A Data Scientist plays games

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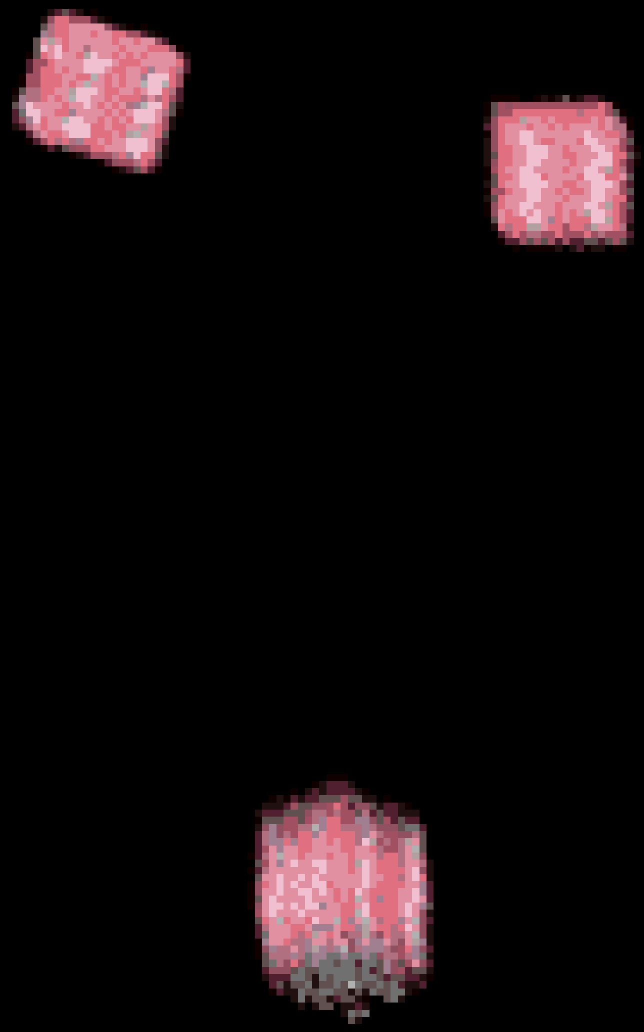
Hi, my name is Nick, and I’m a Data Scientist …

Let’s start with a game …



Roll 1 die

I'll give you $1

You give me $1

1, 2,

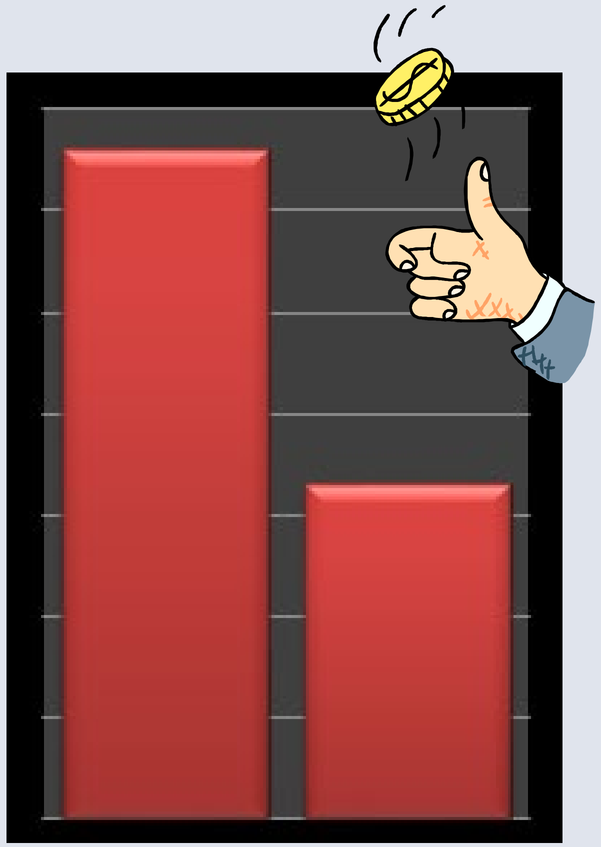
How about now?

3, 4, 5, 6

Would you play this game?

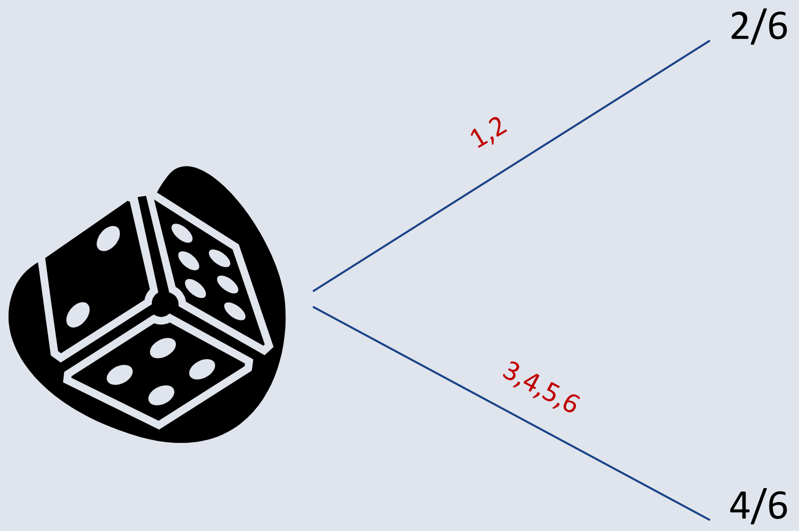
Two basic methods:

Experimentation



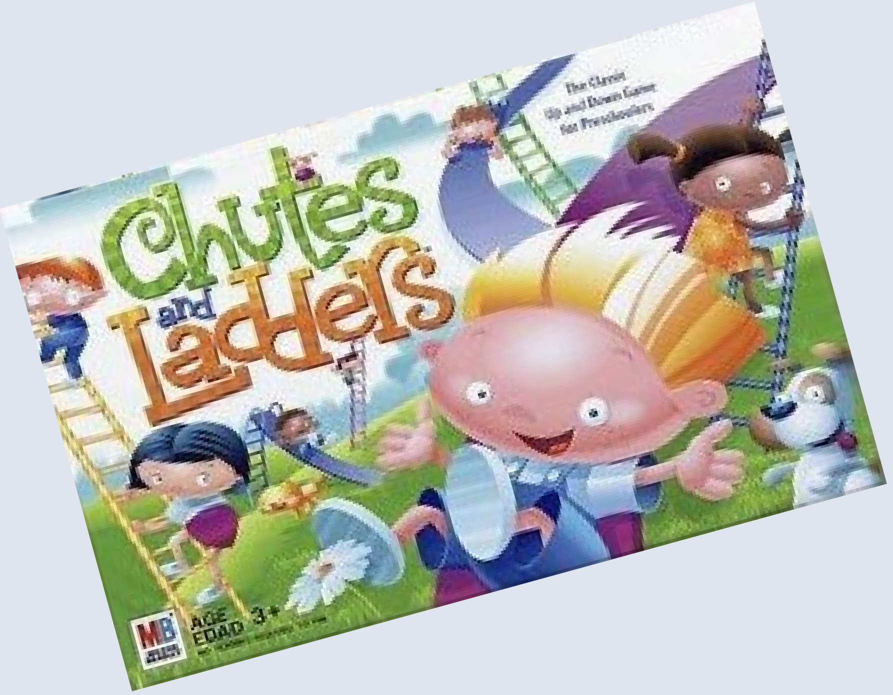
Repeat the same experiment over and over again to compile results.

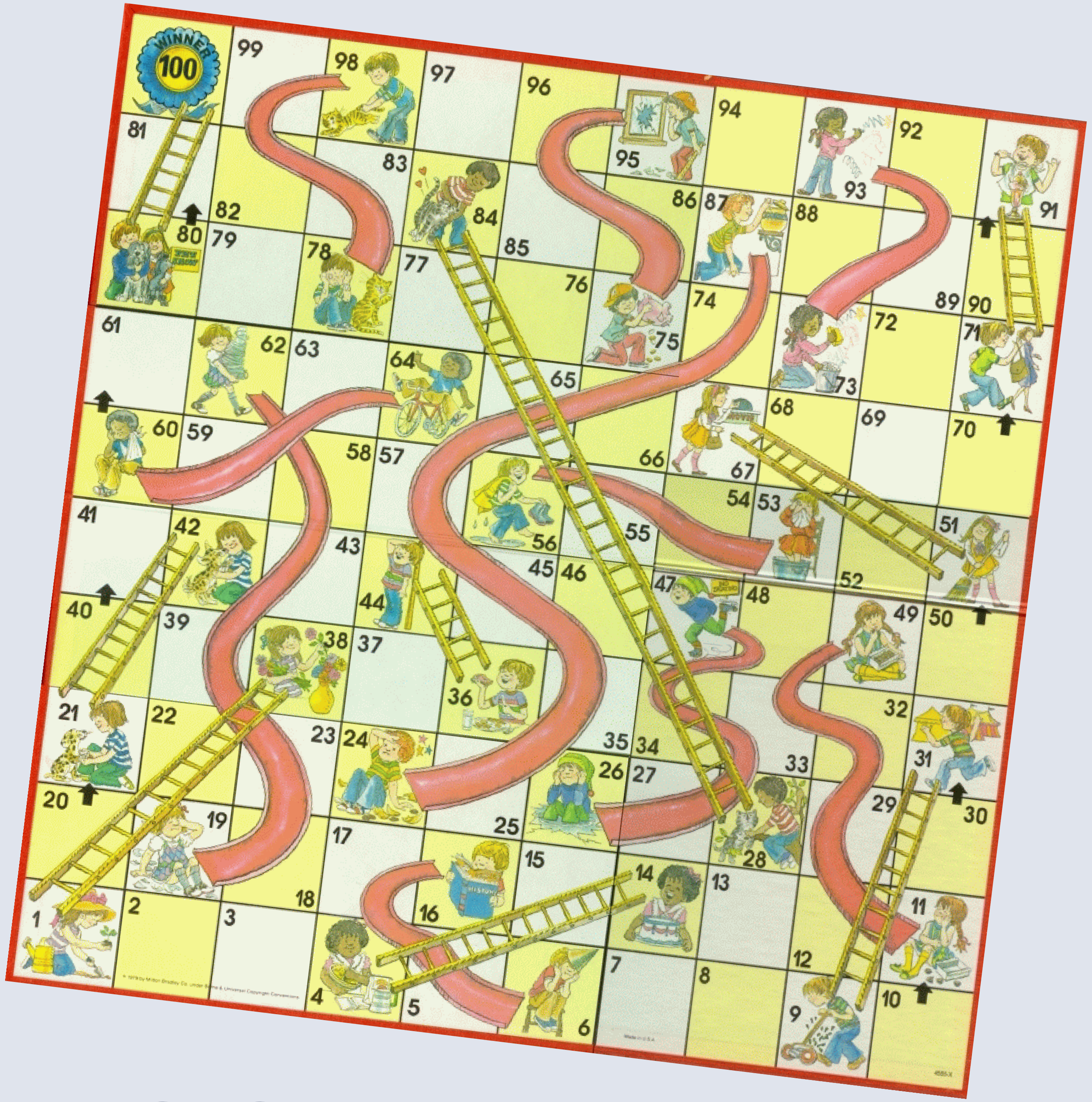
Formal Modeling



Mathematically model and calculate exact probabilities.

Real Game Examples

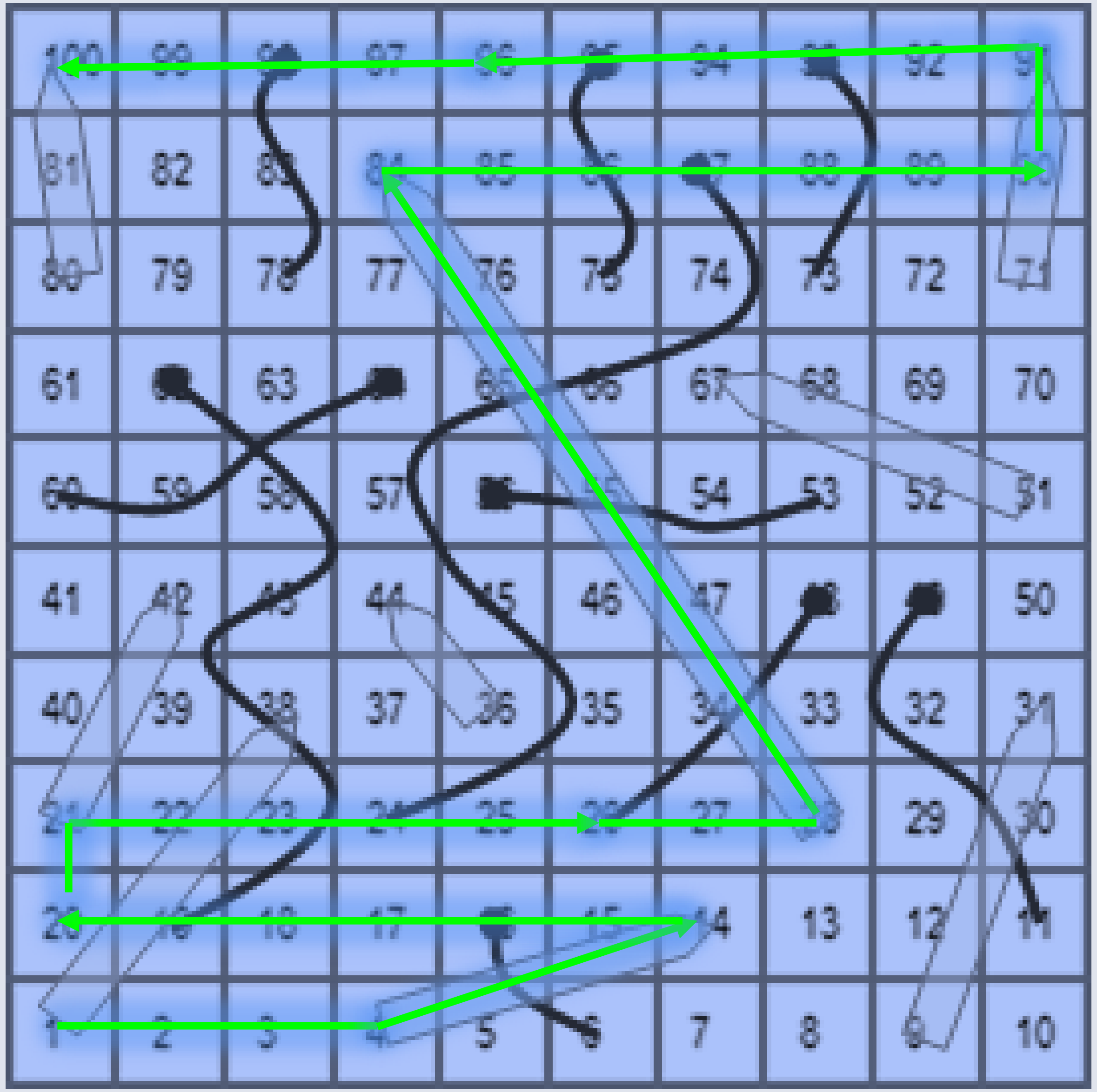




Snakes and Ladders

Win!

How long does a game last?



Directed Graph

The shortest possible game takes just seven rolls.

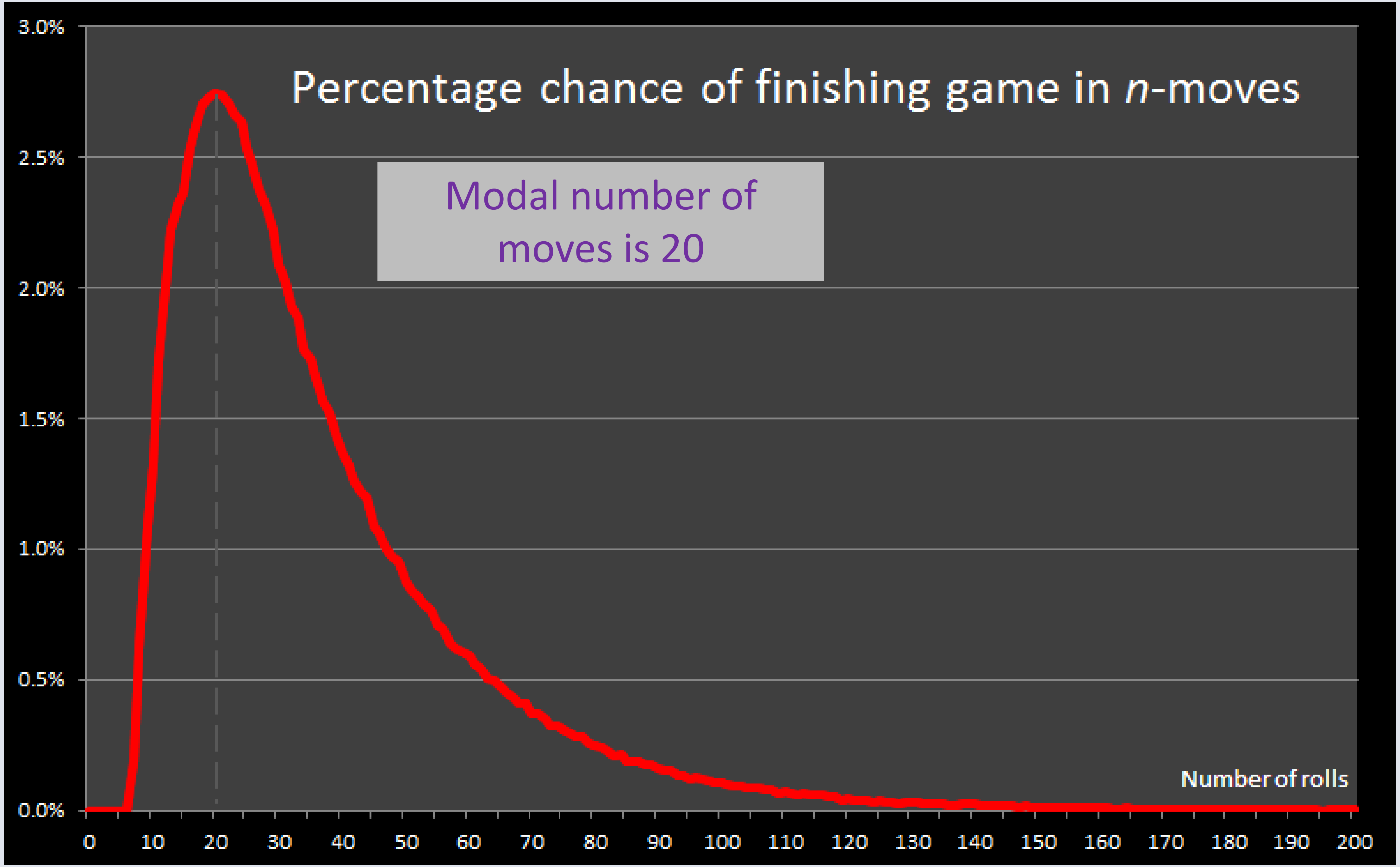
There are multiple ways this can be achieved, it happens approximately twice in every thousand games played.

One possible solution is the rolls:

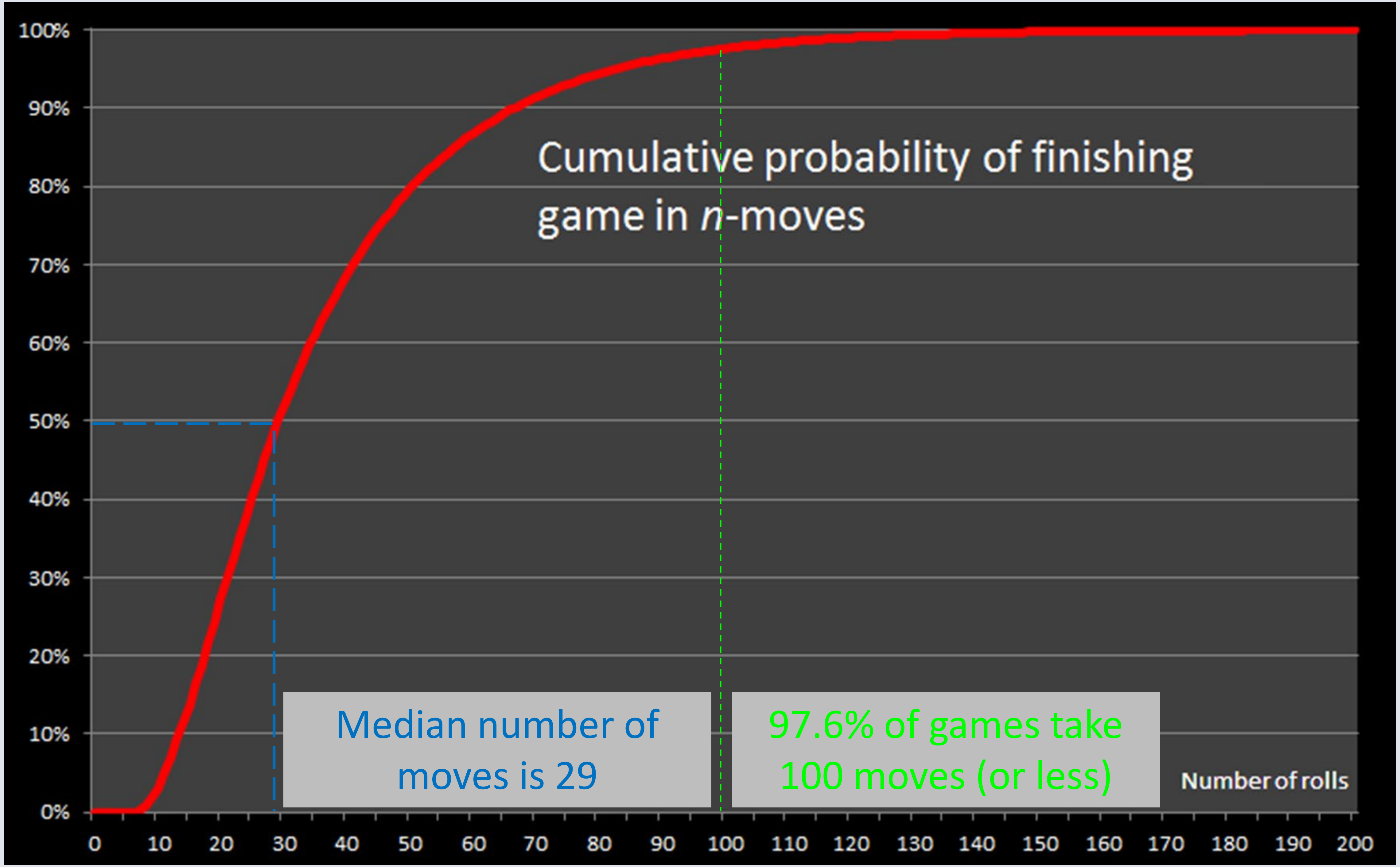
4, 6, 6, 2, 6, 6, 4

Monte-Carlo Simulation

One billion games!



Cumulative chance of winning



What kind of average are you looking for?

- MODAL number of moves = 20
  (Most common number of moves to complete the game)

- MEDIAN number of moves = 29
  (As many games take <u>*less*</u> time to complete as do <u>*more*</u>)

- (Arithmetic) MEAN number of moves = 36.2
  (Sum of all moves divided by number of games, for large N)

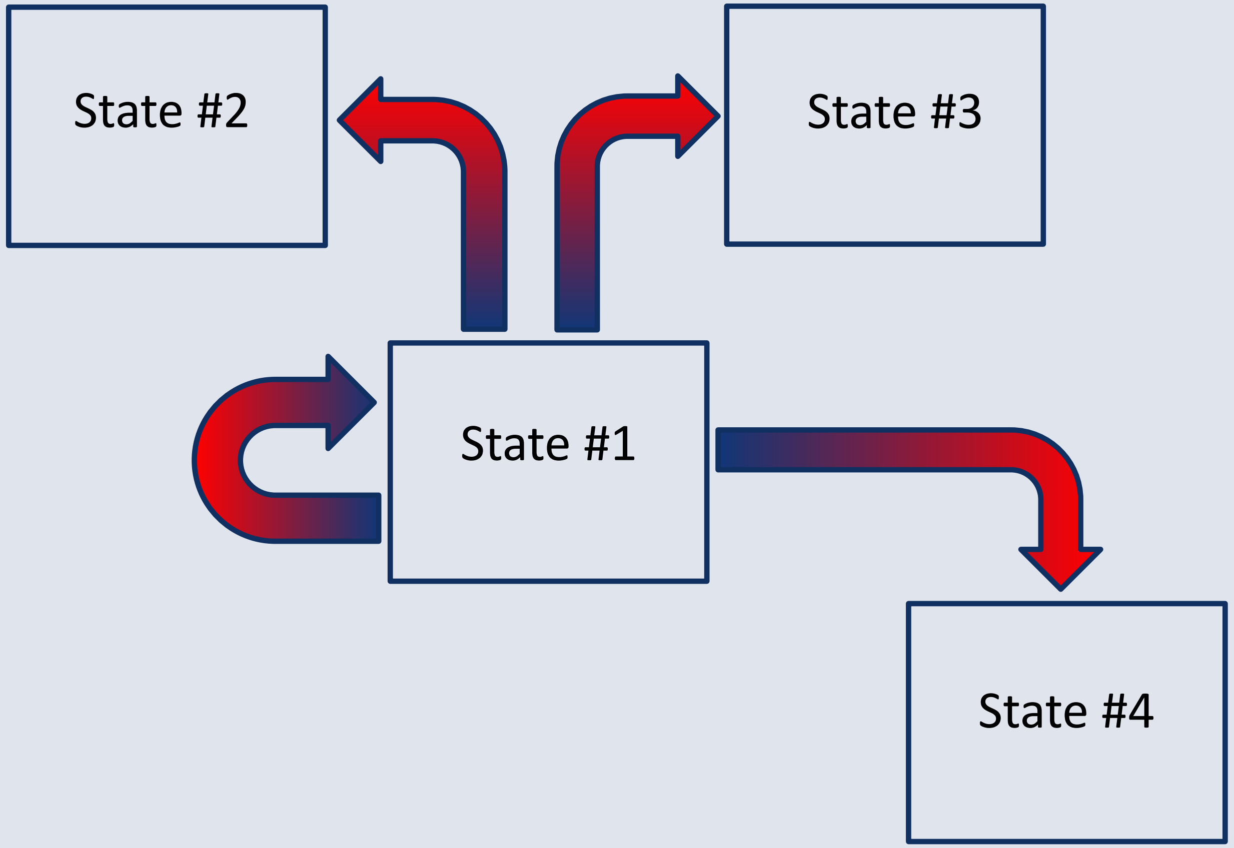
Андре́й Андре́евич Ма́рков

(1856-1922)

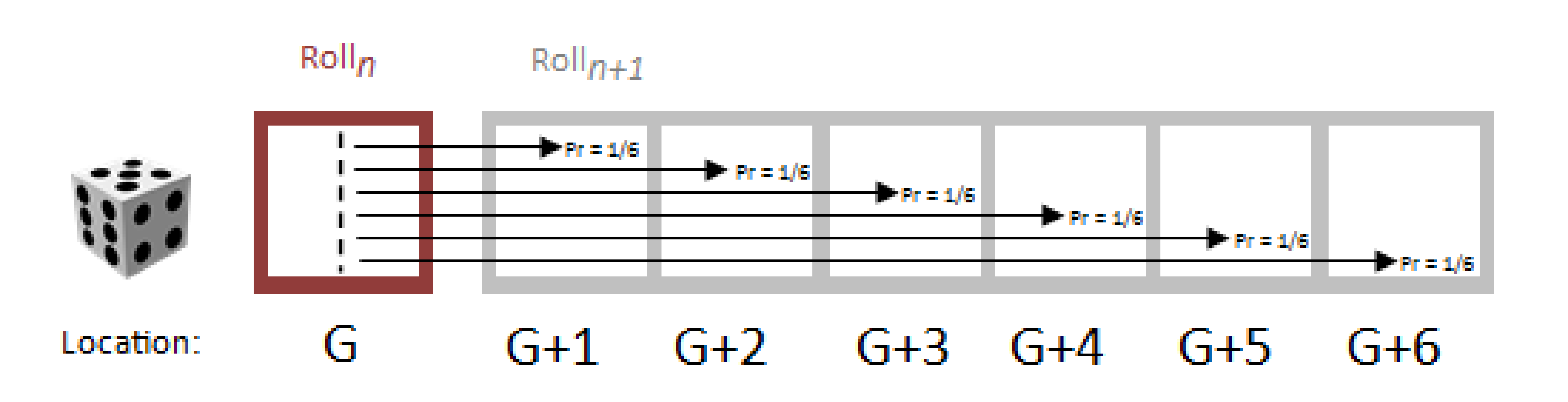
Subjective Approach – Markov Chains

Model a system as a series of states.

Calculate the stochastic probabilities of transitioning from one state to any other.



Stochastic Process



Crucial to this simple analysis is the concept of a *memoryless* system.

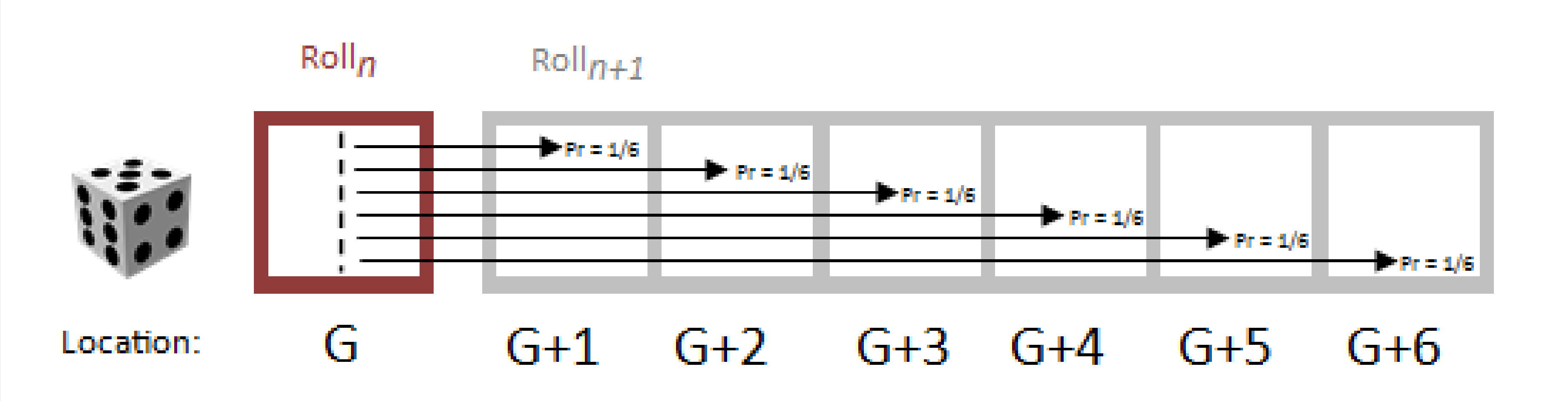
It does not matter *how* we got to square G, but once there, we know the *probabilities* of moving to other squares.

All probabilities *must* add up to 1.0 (something must happen)

$$\begin{array}{cc}
 & \begin{array}{ccccccc} & & & j & & & \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{array}{cc} & 1 \\ & 2 \\ i & 3 \\ & 4 \\ & 5 \\ & 6 \\ & 7 \end{array} &
\begin{pmatrix} & & & & & & \\ & & & & & & \\ & & & a_{i,j} & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{pmatrix}
\end{array}$$

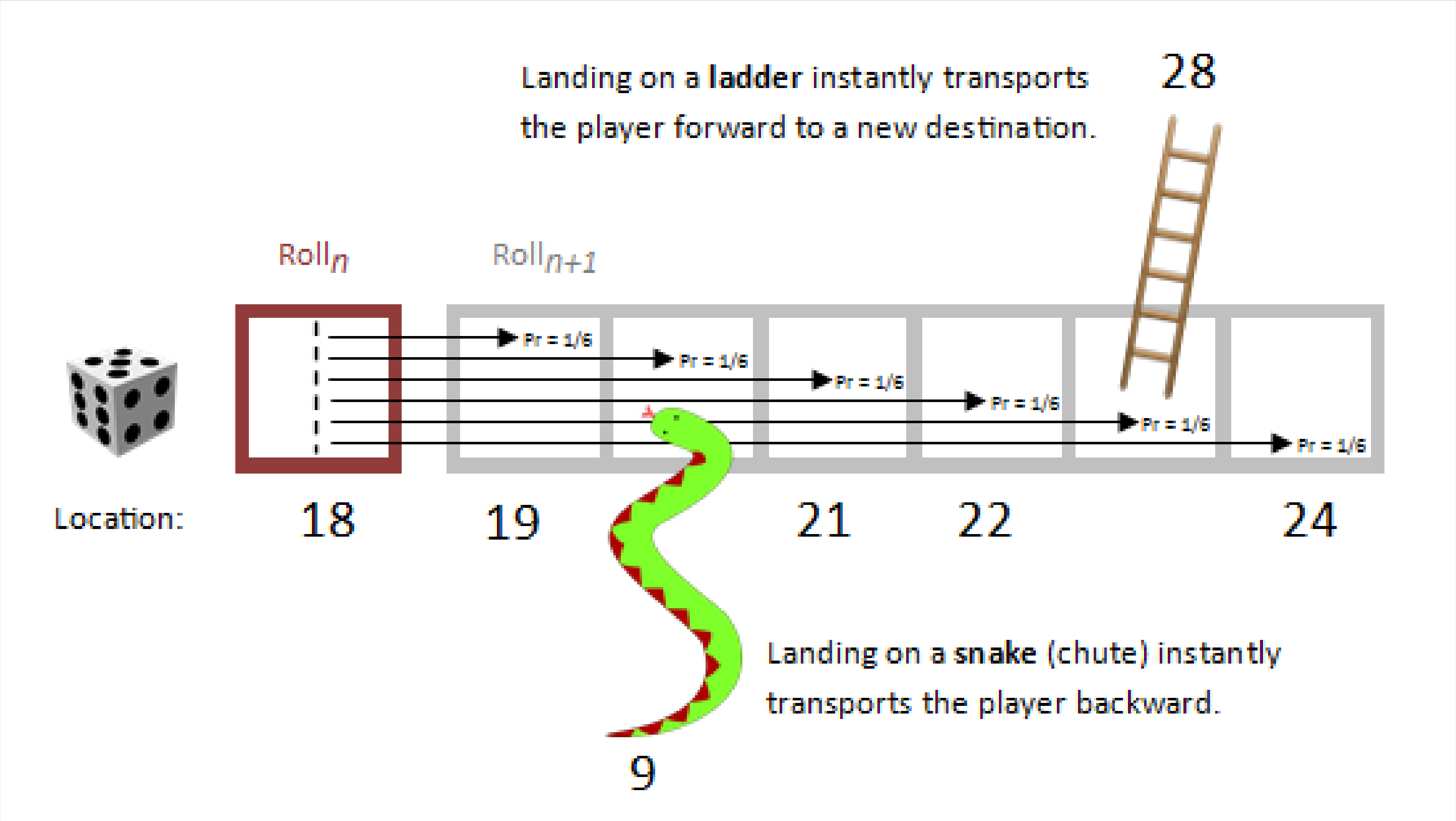
Square matrix containing probabilities of transitioning from state i to state j on next step.

Transition Matrix



(Sparse) matrix containing probabilities of transitioning from state *i* to state *j* on next move

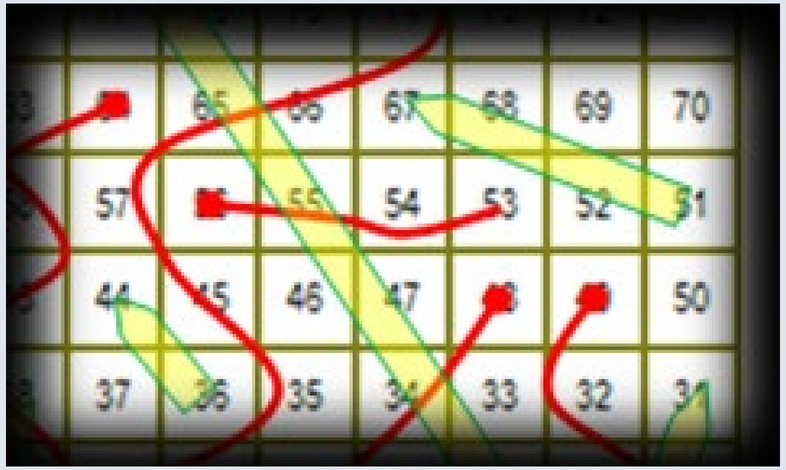
$$
\begin{array}{c}
\\ 10 \\ 11 \\ 12 \\ \\
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc}
 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 &
\end{array} \\
\left(\begin{array}{cccccccccccccc}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}\right)
\end{array}
$$



Snakes and Ladders Transition Matrix

		8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
17	...	0	1/6	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	0	0	0	0	0	1/6	0	0	...
18	...	0	1/6	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	0	1/6	0	0	0	1/6	0	0	...
19	...	0	1/6	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	0	0	1/6	0	0	...

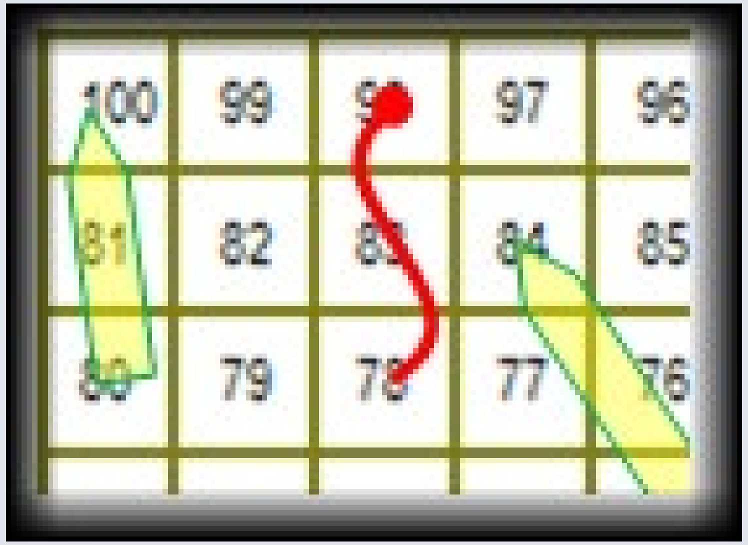
Watch out *! #1*



Some squares you can get to in more than one way!

$$
\begin{array}{c}
\begin{array}{ccccccccccccccccccccccccc}
 & 48 & 49 & 50 & 51 & 52 & 53 & 54 & 55 & 56 & 57 & 58 & 59 & 60 & 61 & 62 & 63 & 64 & 65 & 66 & 67 & 68 & 69 & 70 &
\end{array}\\
50\;\begin{pmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & 0 & 0 & 0 & 0 & 1/6 & 2/6 & 1/6 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 0 & 0 & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}
\end{array}
$$

Watch out *! #2*



When you get to the end of the game, you don't need an exact roll to finish.

$$\begin{array}{cc} & \begin{array}{ccccccccccccccccccccccccc} & 77 & 78 & 79 & 80 & 81 & 82 & 83 & 84 & 85 & 86 & 87 & 88 & 89 & 90 & 91 & 92 & 93 & 94 & 95 & 96 & 97 & 98 & 99 & 100 \end{array} \\ \begin{array}{c} \\ 96 \\ 97 \\ 98 \\ \\ \end{array} & \left(\begin{array}{ccccccccccccccccccccccccc} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & 0 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 0 & 1/6 & 3/6 \\ \cdots & 0 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 4/6 \\ \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right) \end{array}$$

Transition Matrix in Action

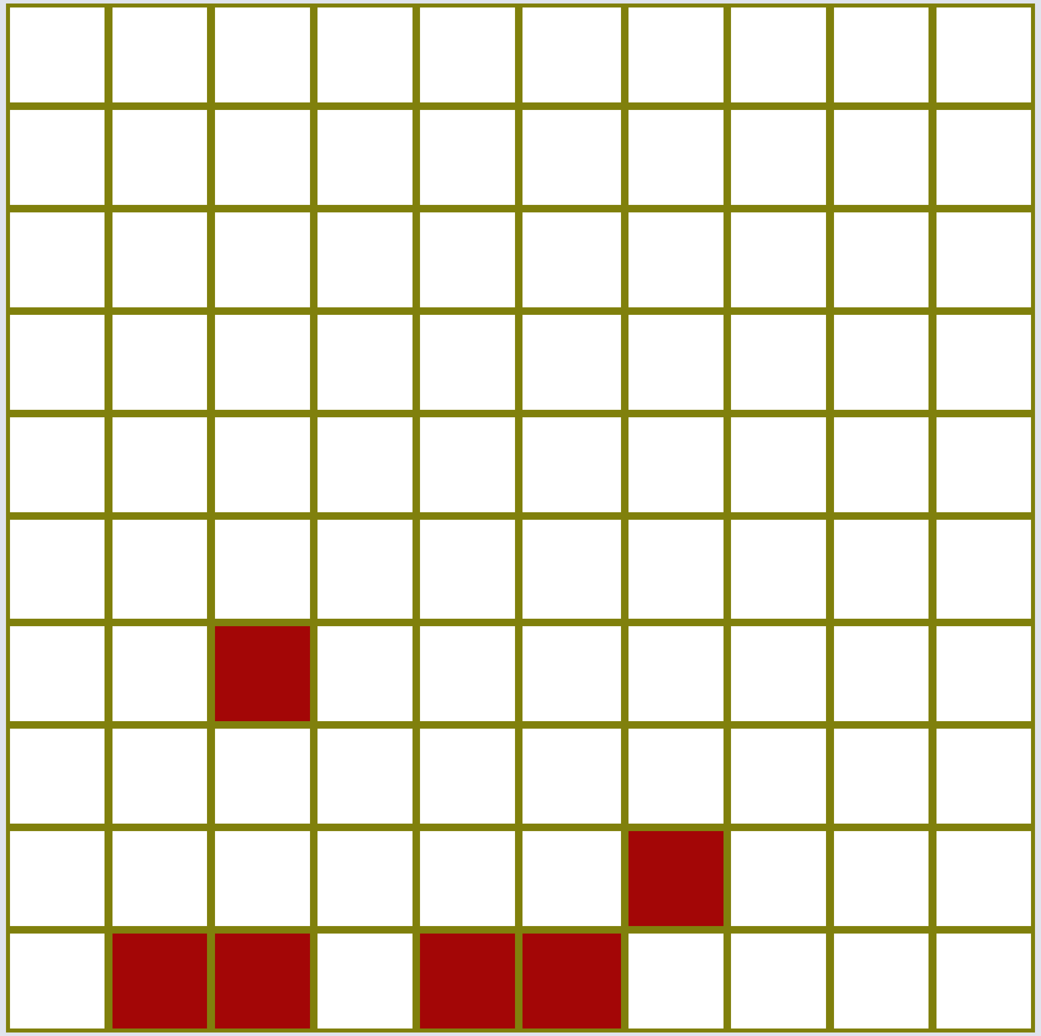
$$\text{Starting States} \times \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & \cdots & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & \cdots & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & a_{i,j} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m,1} & a_{m,2} & \cdots & \cdots & \cdots & a_{m,n} \end{bmatrix} = \text{Ending States}$$

Results – Roll #1

Create a column vector with 1.0 in location *i*=0

(Player starts at state zero, off the board)

Multiply this by the Transition Matrix



Output row vector shows probability of where player could be after one roll

Wash, Rinse, Repeat

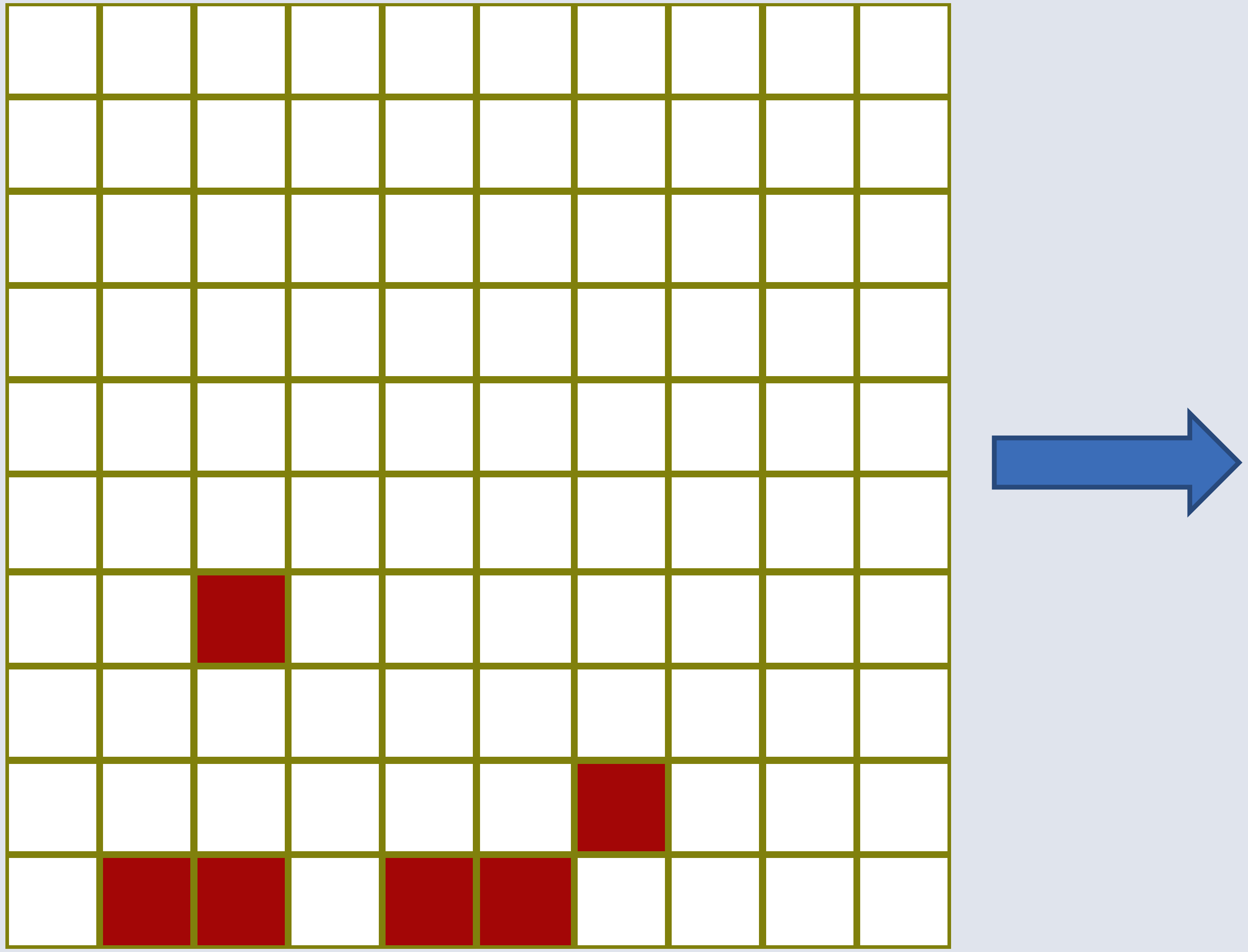
Starting States

$$\times \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & \cdots & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & \cdots & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & a_{i,j} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m,1} & a_{m,2} & \cdots & \cdots & \cdots & a_{m,n} \end{bmatrix} =$$

New Ending States
Ending States

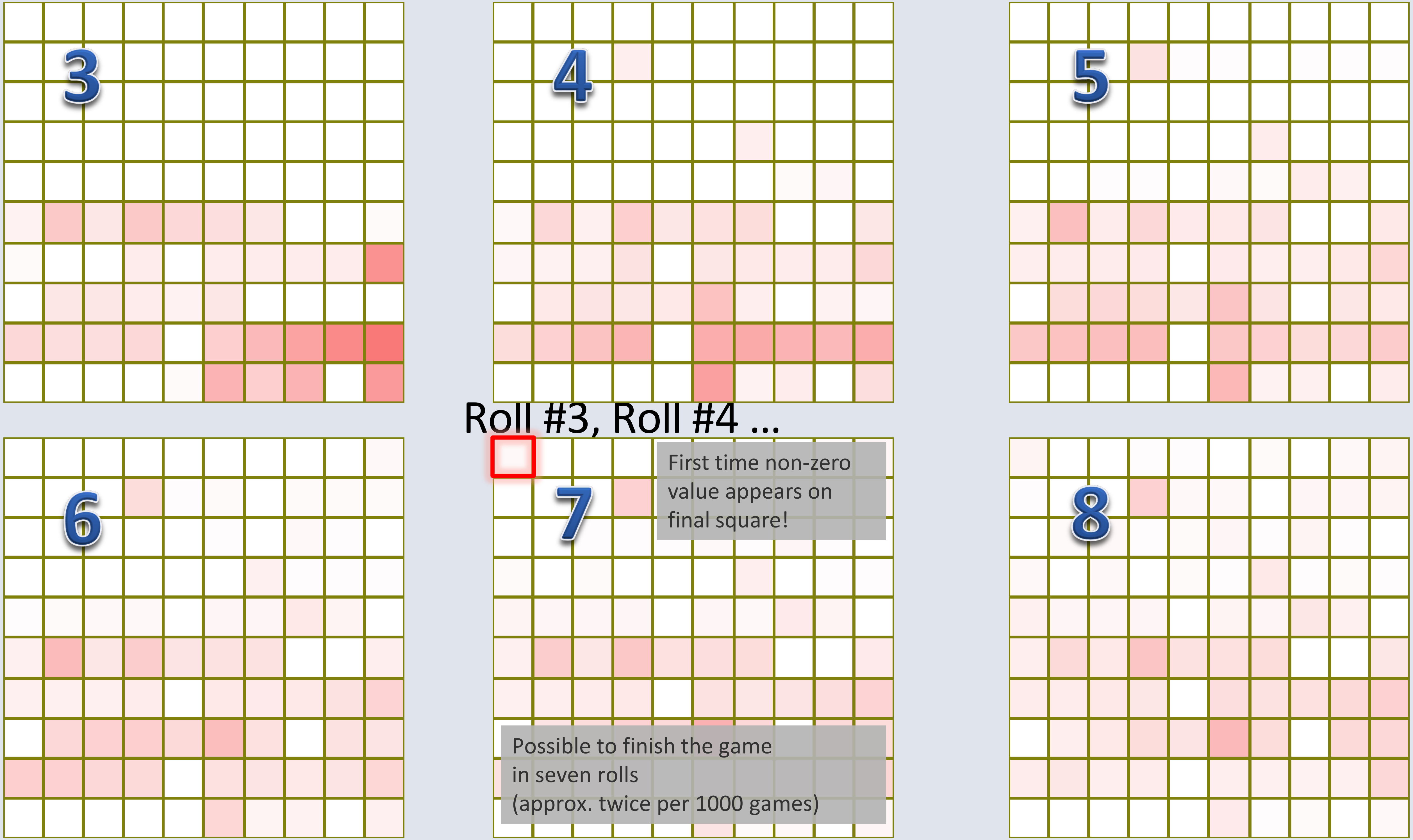
Roll #2

Now use the probability output from roll #1 as the input for roll #2, and multiply by the Transition Matrix again.

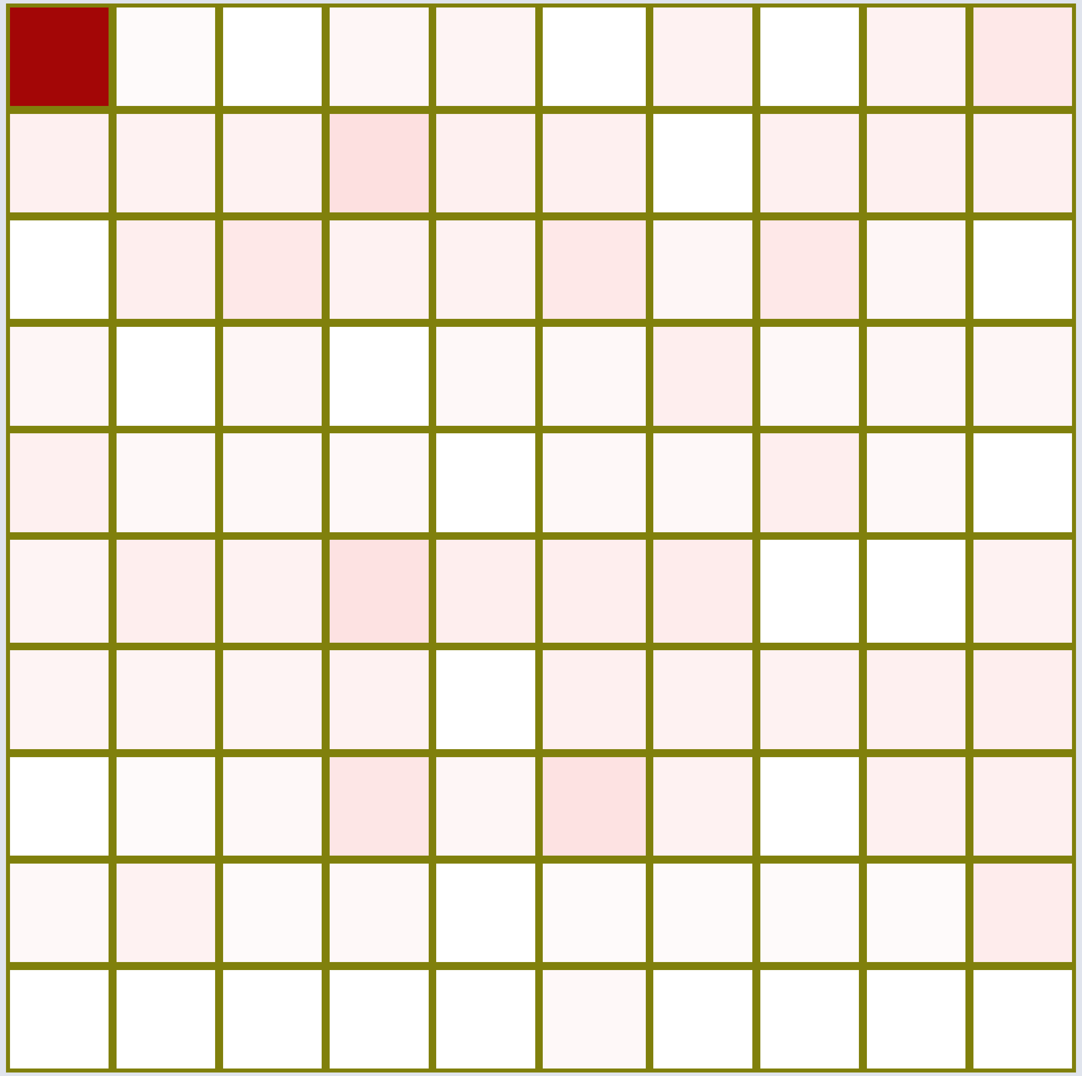


Roll #1

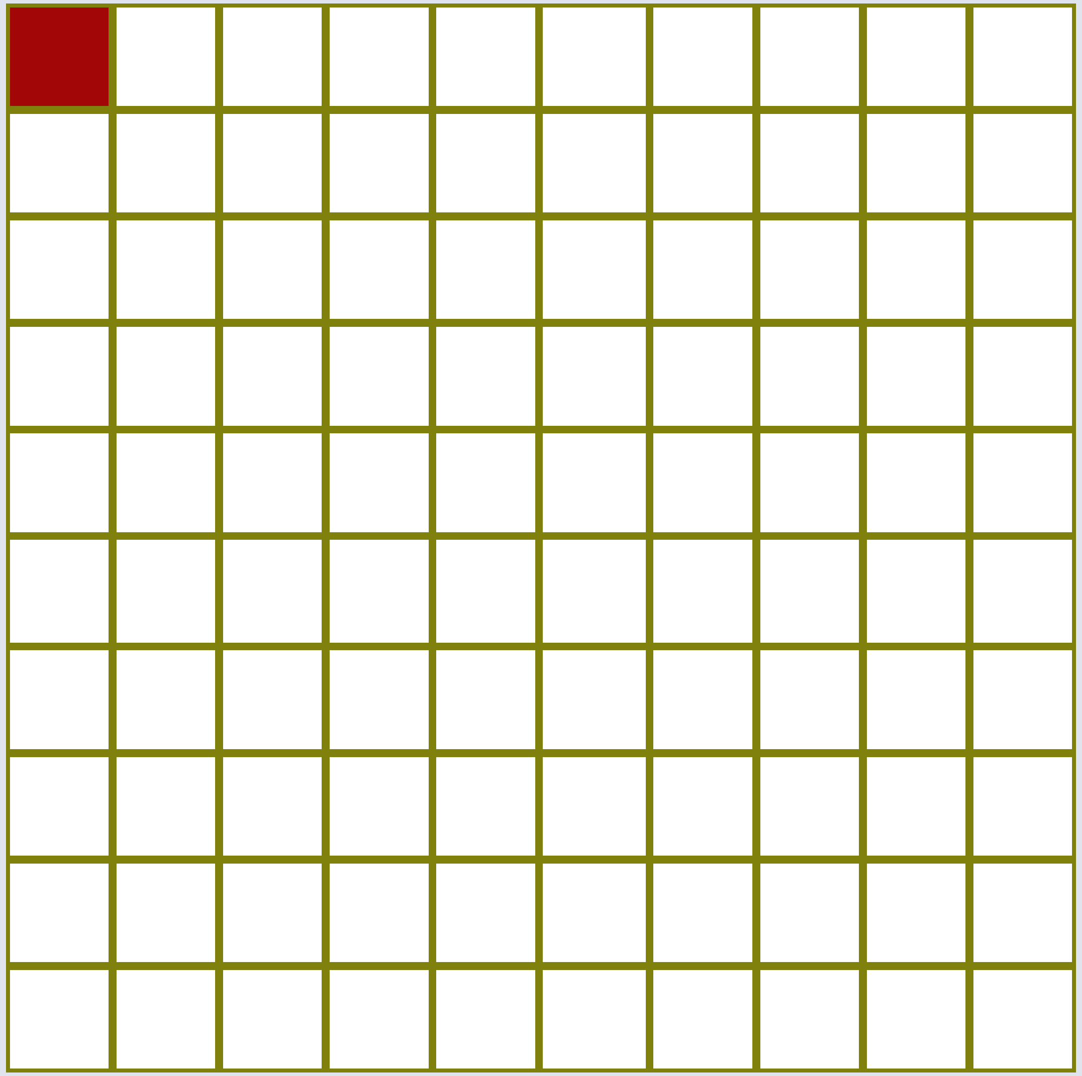
Roll #2


3
4
5
6
7
8
Roll #3, Roll #4 …
First time non-zero value appears on final square!
Possible to finish the game in seven rolls (approx. twice per 1000 games)

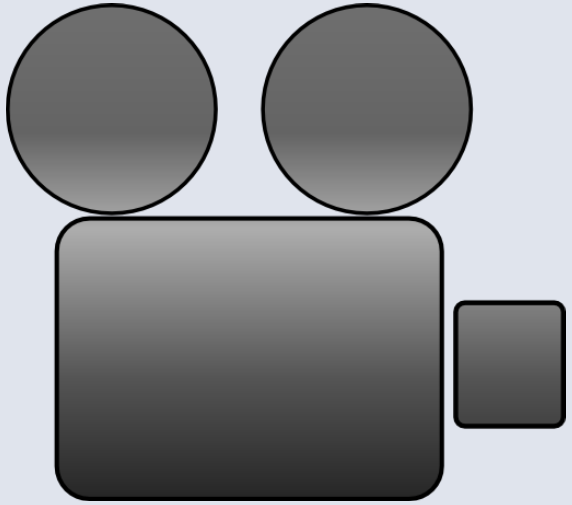
Roll #20, Roll #100

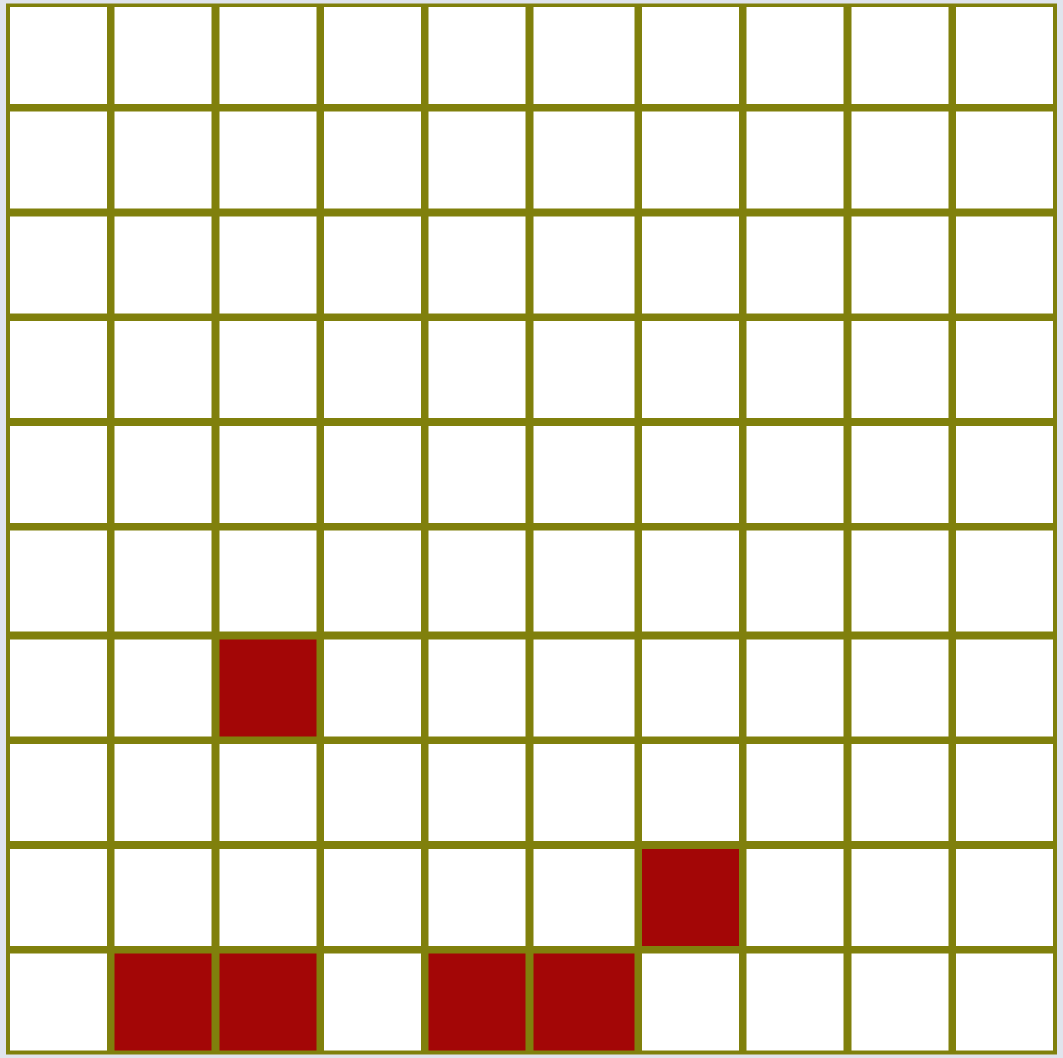


Roll #20

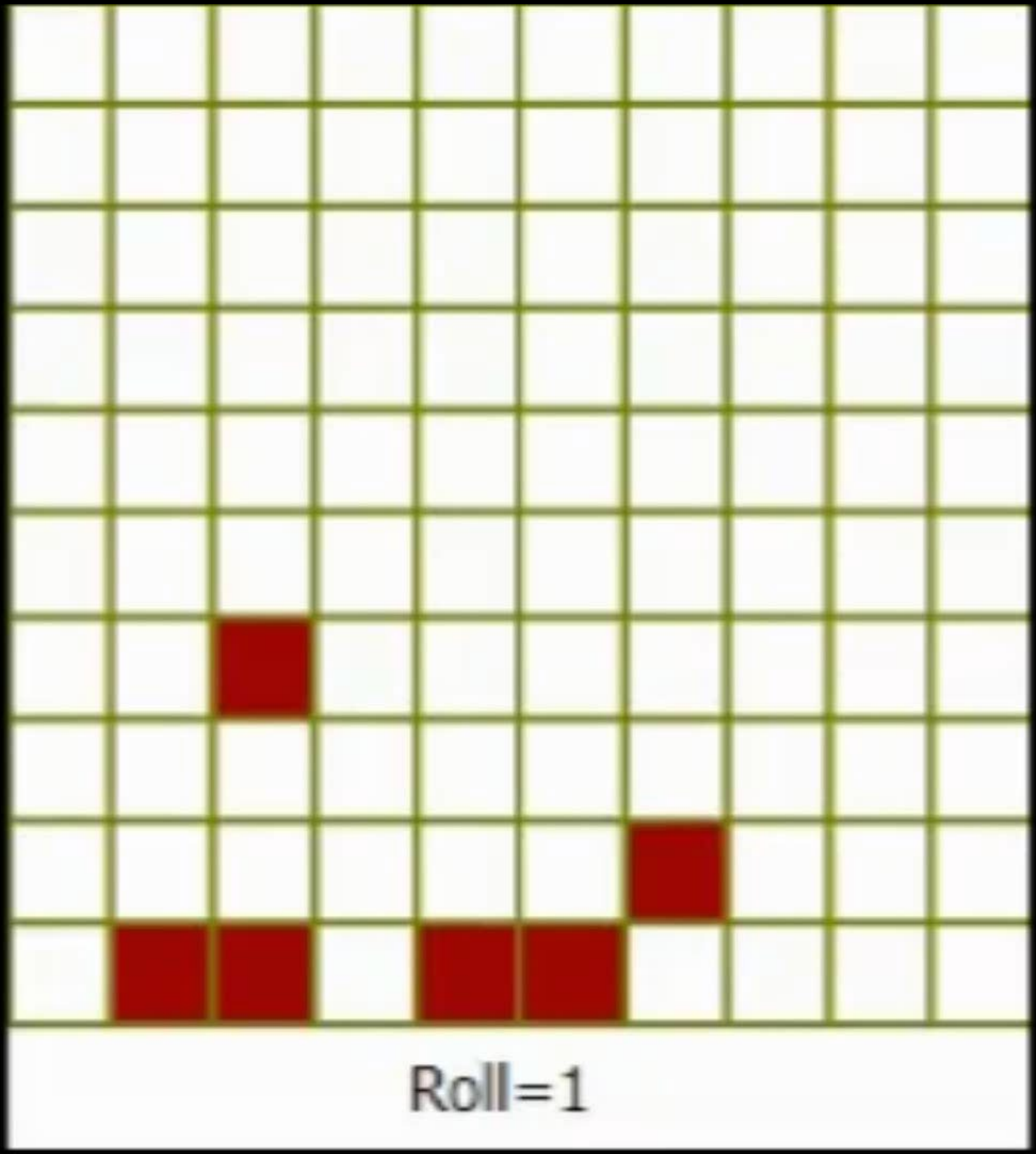


Roll #100

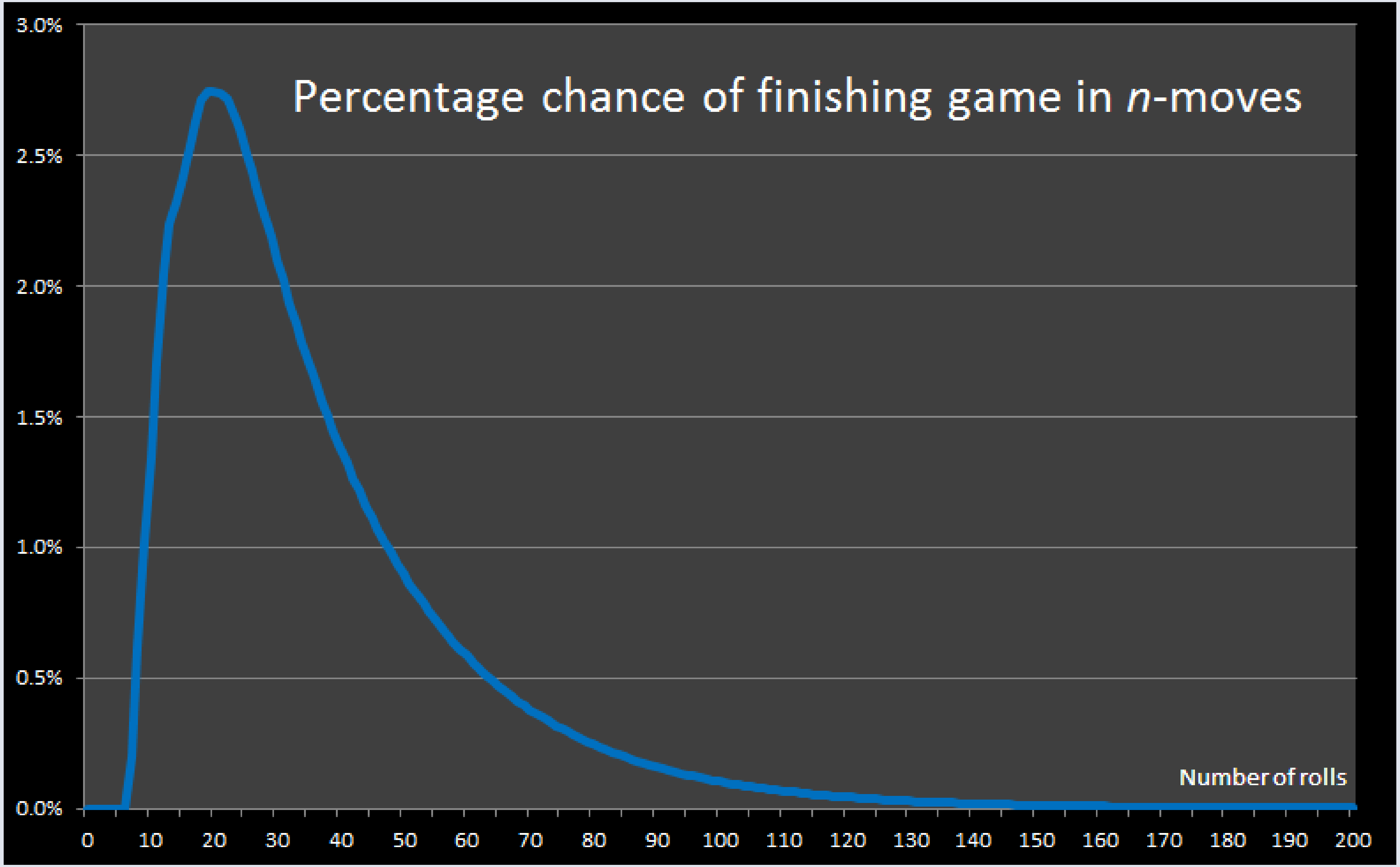


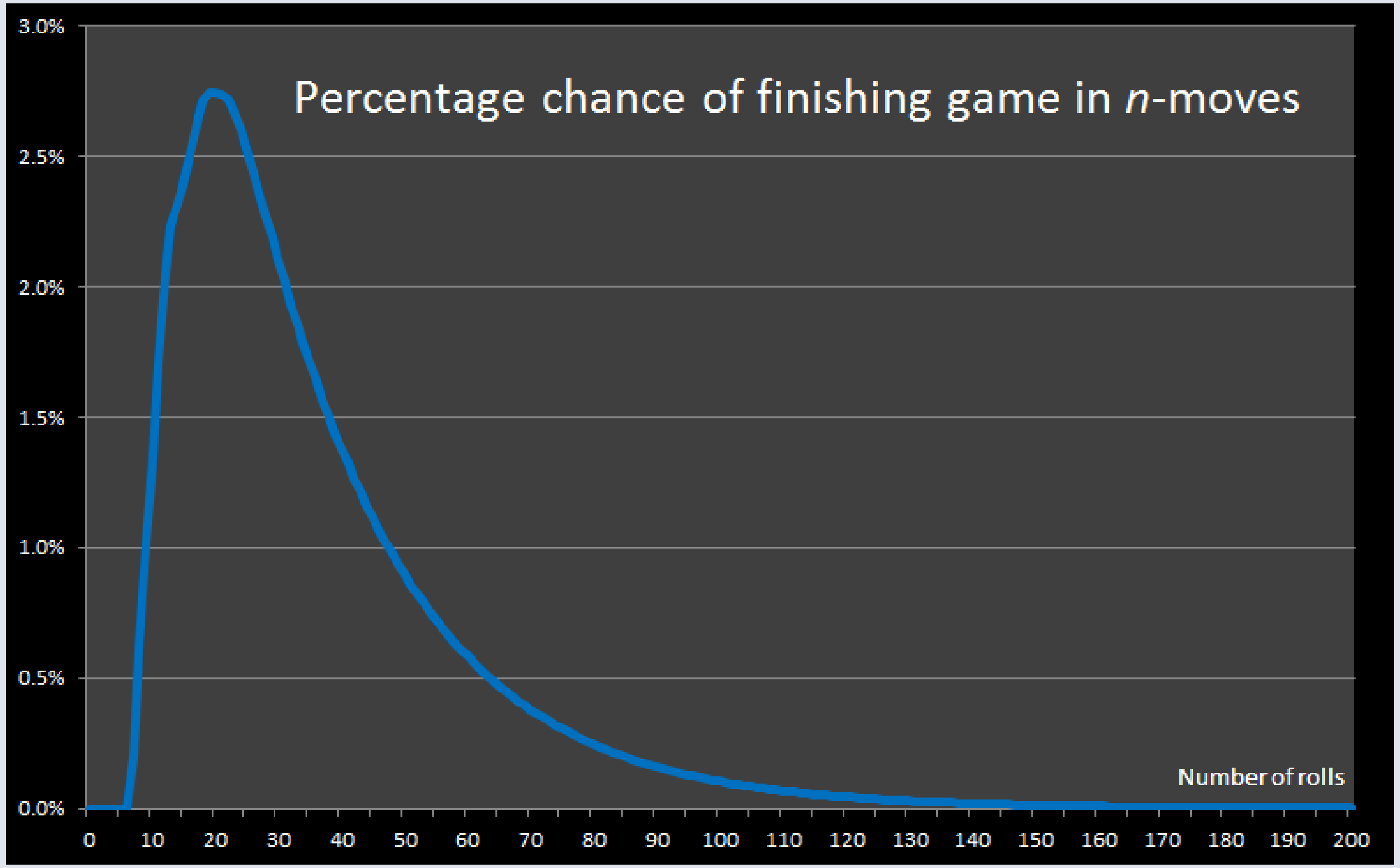


Animation



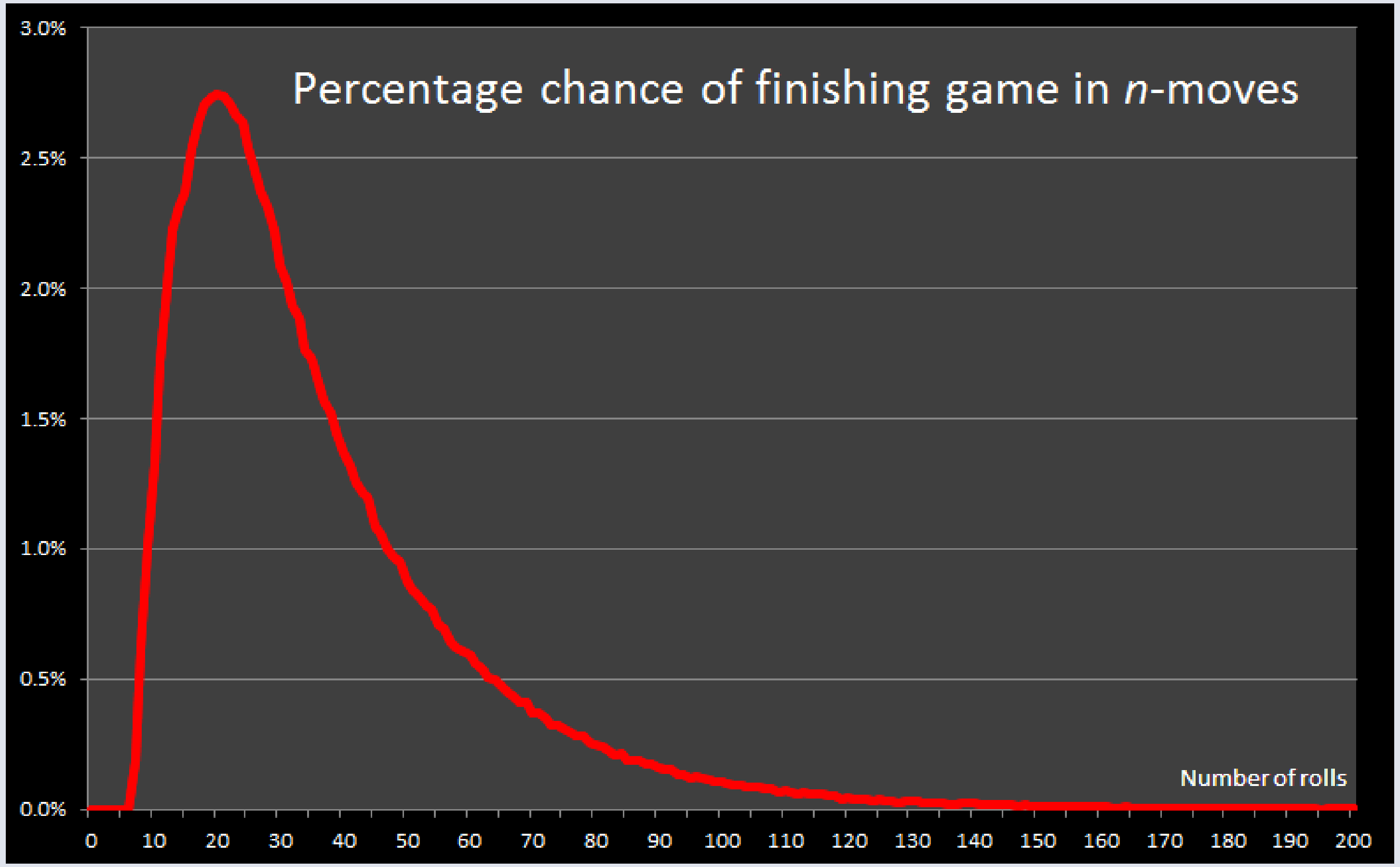
Markov Chain Analysis Results



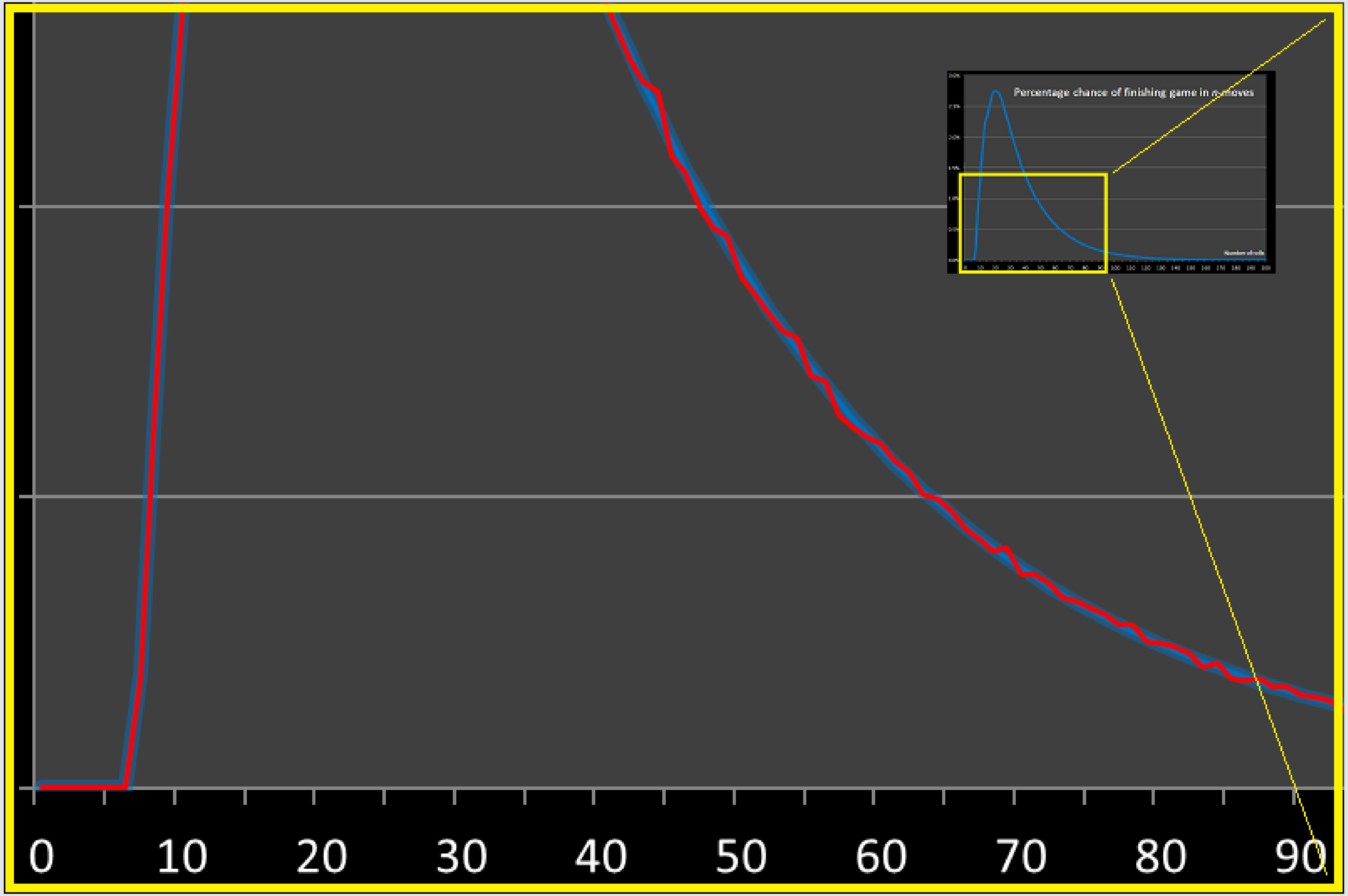


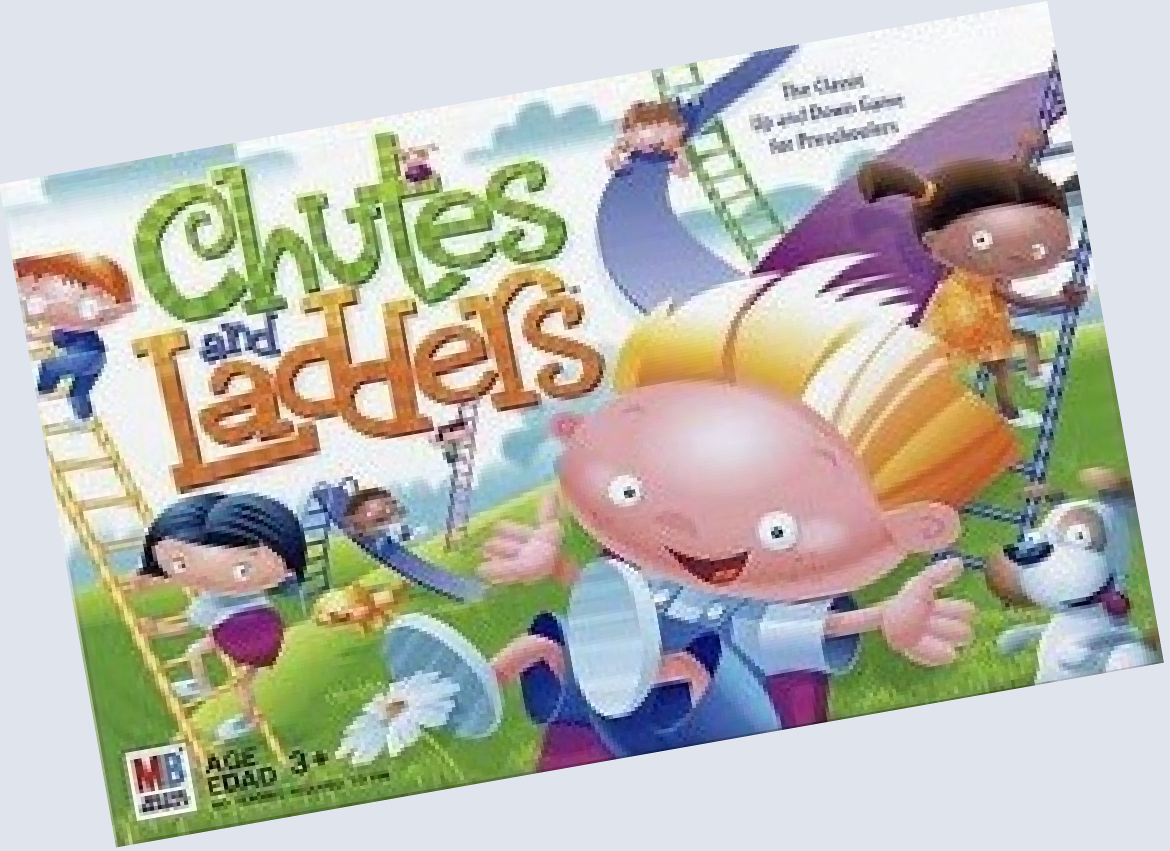
Formal model

Experimentation



Comparison of methods





Paramapada Sopanam — *"The Ladder to Salvation."*

2nd Century *B.C.*

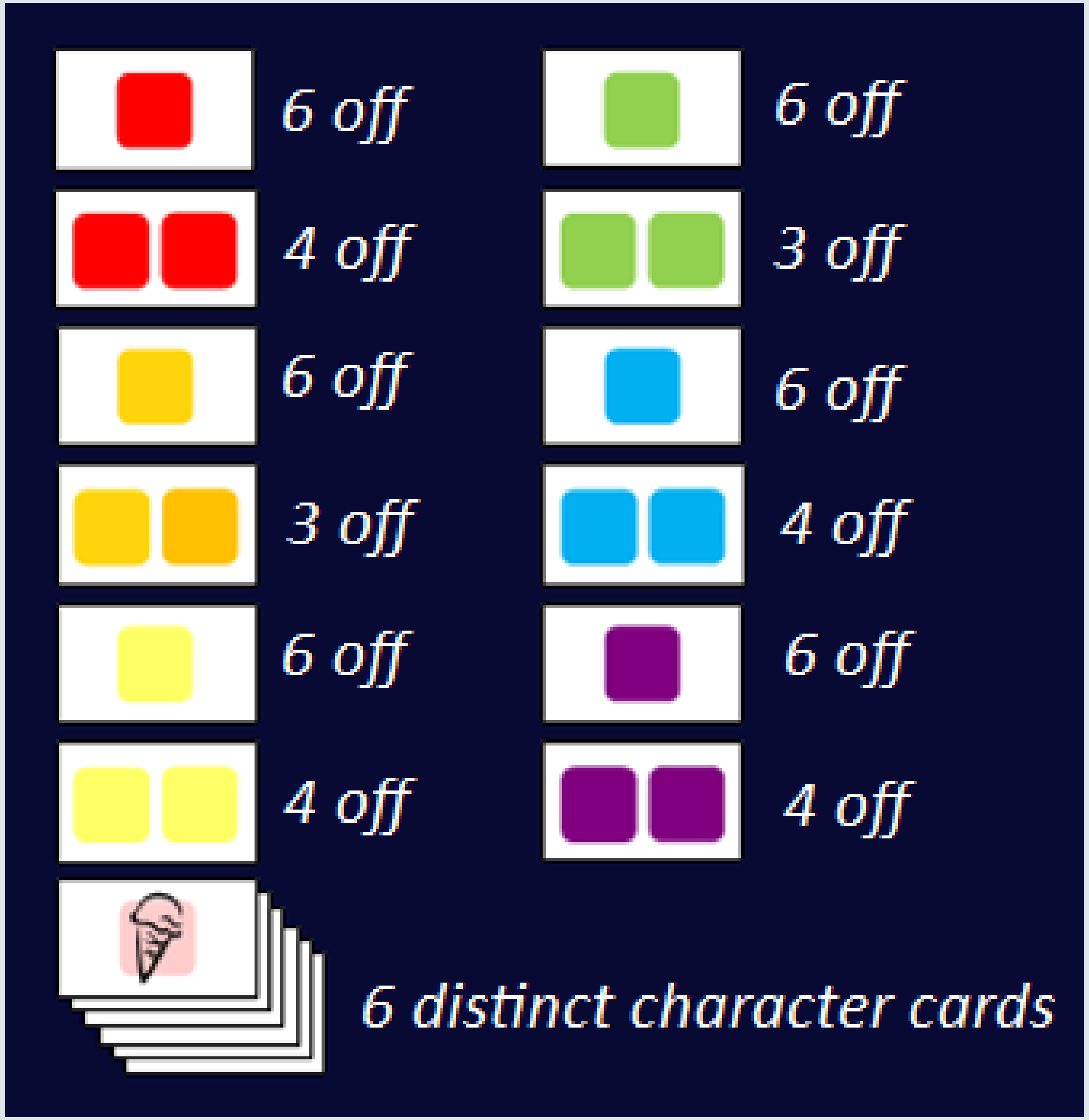
It was invented by Hindu spiritual leaders to teach children about the rewards of good deeds and the negative consequences of bad ones.

- Snakes represent vices and poor choices.
- Ladders represent virtues and sound morality
- Square 100 is “Nirvana”


CANDY
LAND


CANDY LAND

Uh-oh! Not a *memoryless* system



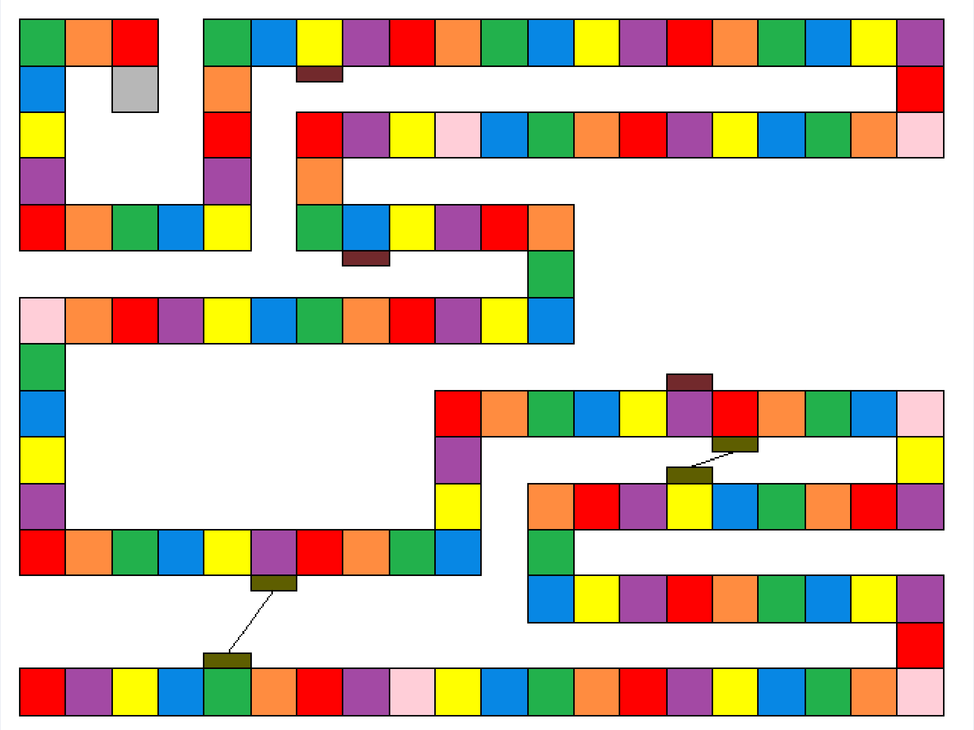
Cards are drawn from a deck and then discarded.

Probability of drawing the next card depends on cards already drawn (Like playing Blackjack).

Crippled Markov Chain

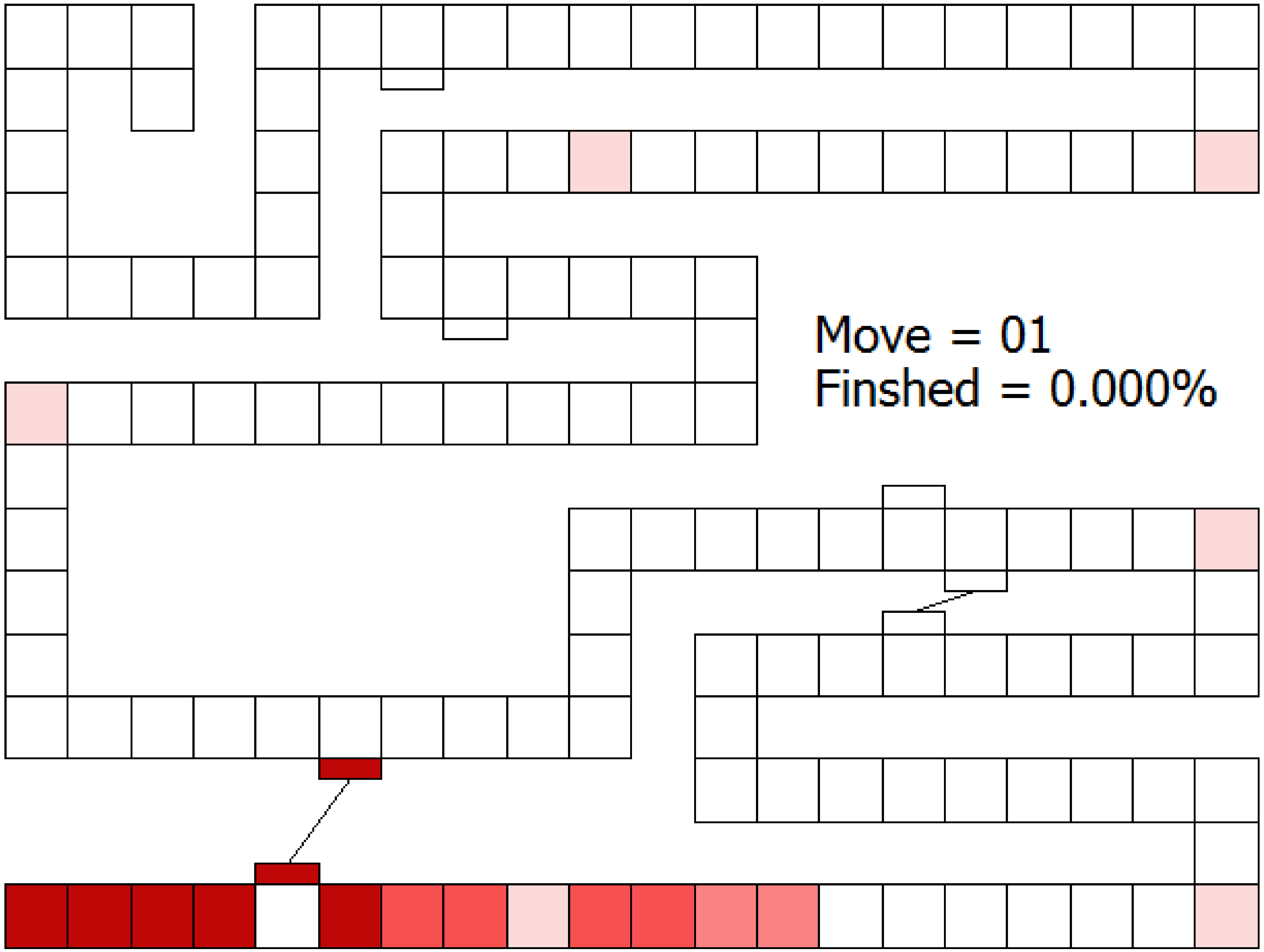
Approximate system by drawing a card, acting on it, then inserting back into deck, shuffling and then drawing again.

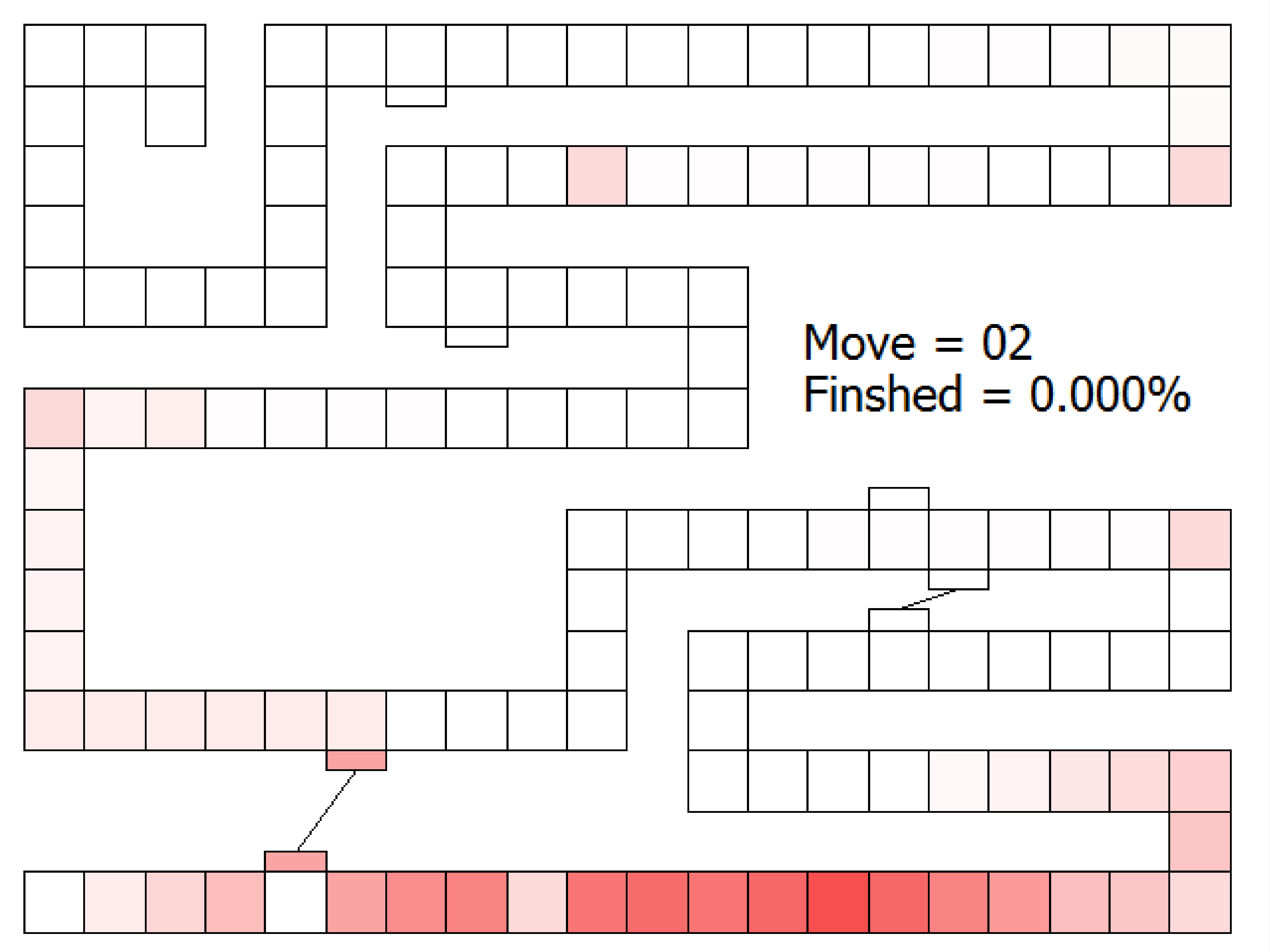
Transition Matrix is easy to create based on relative distributions of cards in the deck.

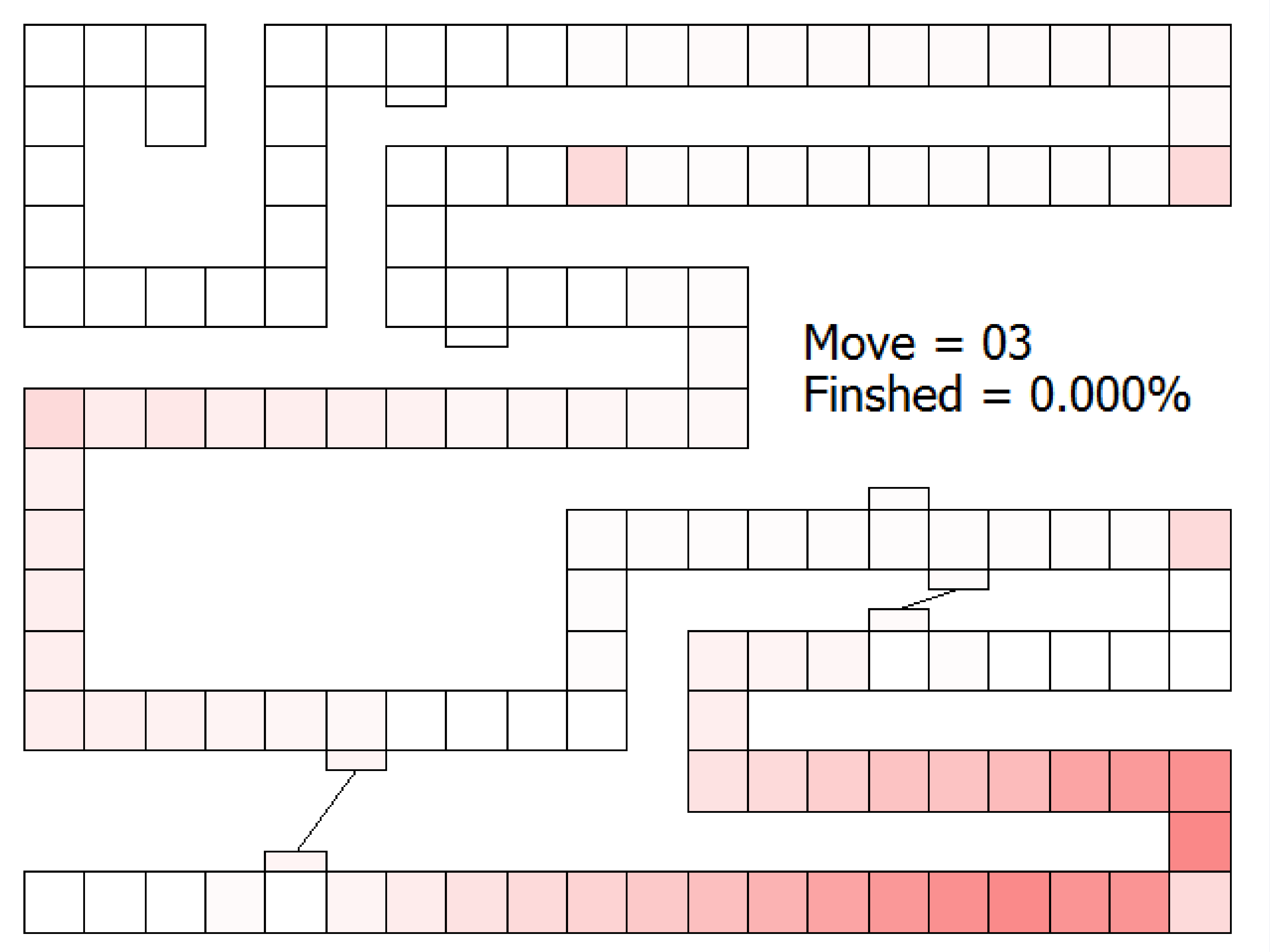


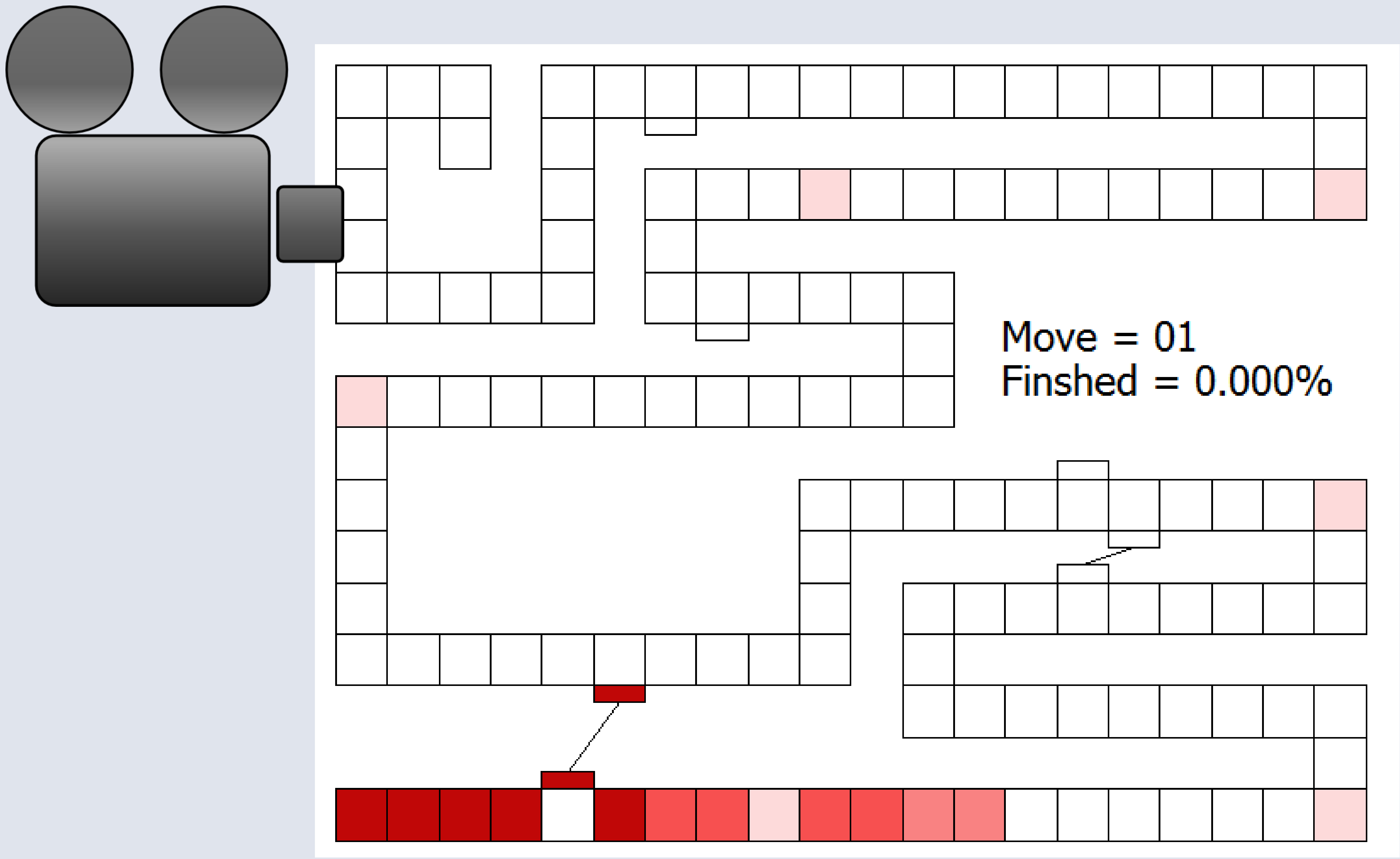
$$\begin{array}{cc}
 & \begin{matrix} & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & .. & 40 & 41 & 42 & \cdots \end{matrix} \\
7 & \begin{pmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & .. & \cdots & \cdots & \cdots & \cdots \\
\cdots & 0 & 0 & 0 & 6/64 & 1/64 & 6/64 & 6/64 & 6/64 & 6/64 & 6/64 & 4/64 & 4/64 & 4/64 & 3/64 & 3/64 & 1/64 & 4/64 & 0 & .. & 0 & 1/64 & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & .. & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}
\end{array}$$

Bridges act like ‘ladders’

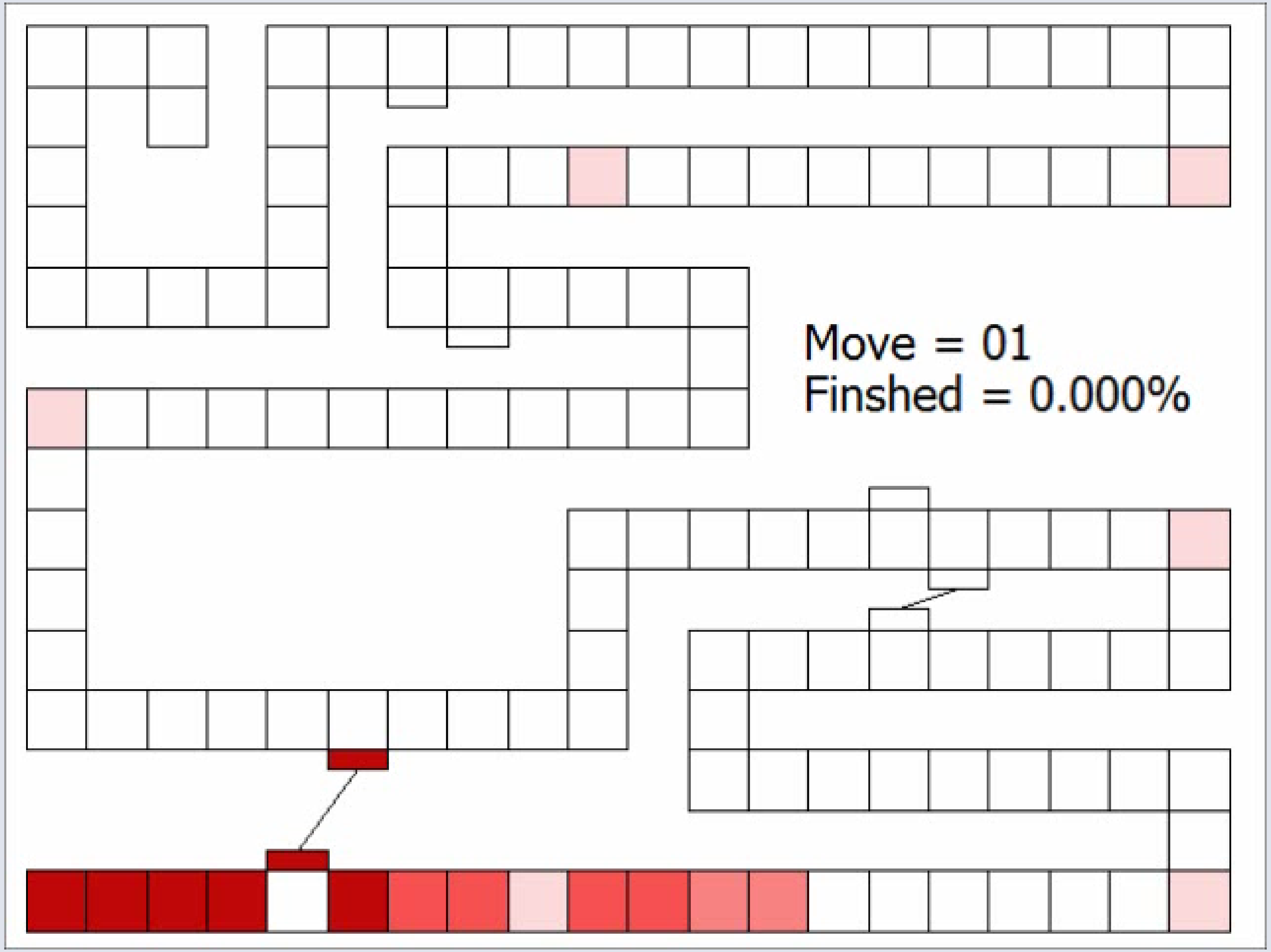

Move = 01
Finshed = 0.000%


Move = 02
Finshed = 0.000%

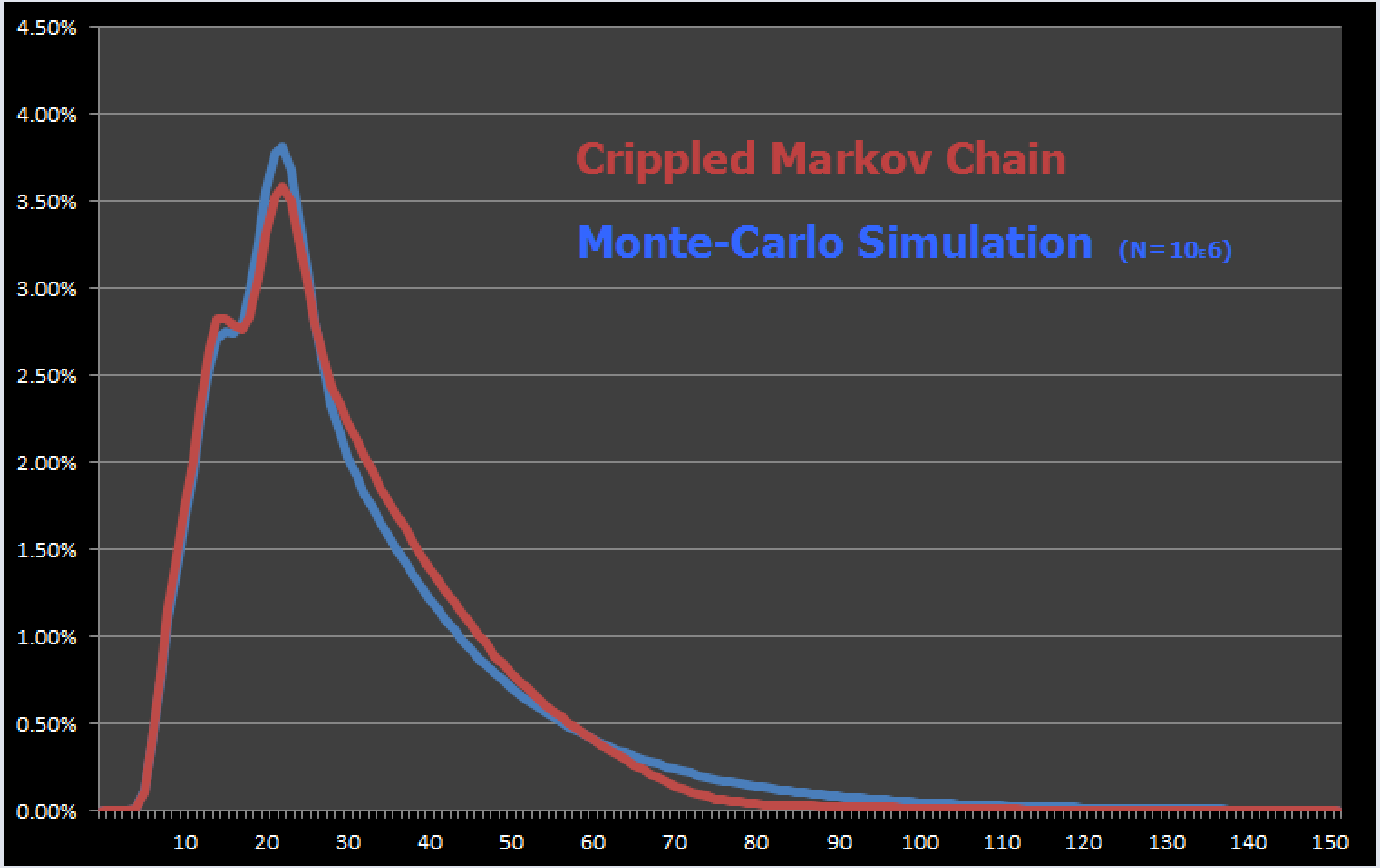

Move = 03
Finshed = 0.000%


Move = 01
Finshed = 0.000%

Animation



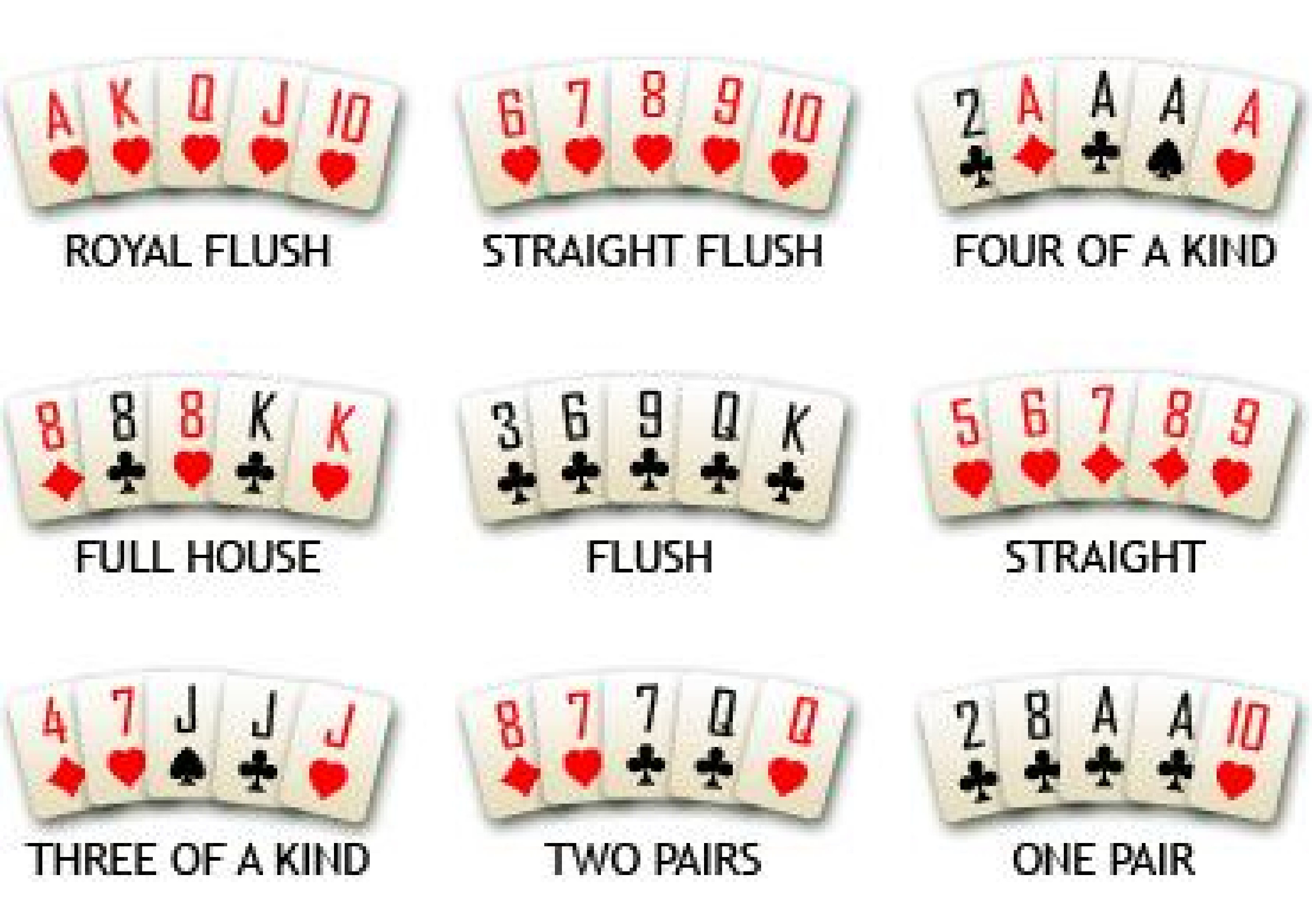
Comparison to Monte-Carlo



Texas Hold'em Poker


What is the best starting hand?

Poker odds are complex



Expected outcome is based on superposition off odds of making each different kind of hand against all possible combinations of opponents hole cards against all combinations of community cards!

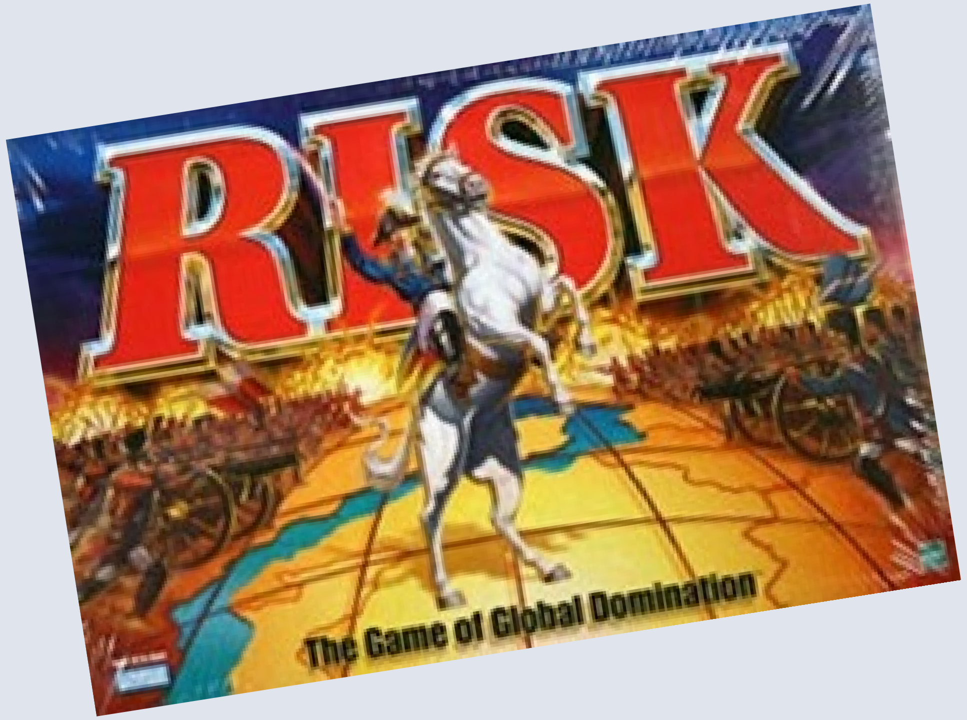
The odds change depending on the number of people at the table!

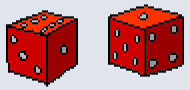
2 PLAYERS

AA #1	AKs #8	AQs #10	AJs #12	ATs #14	A9s #19	A8s #22	A7s #27	A6s #33	A5s #34	A4s #37	A3s #41	A2s #48
AK #11	KK #2	KQs #17	KJs #20	KTs #21	K9s #29	K8s #39	K7s #47	K6s #50	K5s #57	K4s #62	K3s #67	K2s #71
AQ #13	KQ #23	QQ #3	QJs #28	QTs #31	Q9s #43	Q8s #53	Q7s #63	Q6s #69	Q5s #74	Q4s #79	Q3s #85	Q2s #89
AJ #16	KJ #26	QJ #35	JJ #4	JTs #38	J9s #51	J8s #64	J7s #75	J6s #86	J5s #92	J4s #97	J3s #100	J2s #104
AT #18	KT #30	QT #44	JT #52	TT #5	T9s #59	T8s #72	T7s #84	T6s #94	T5s #106	T4s #109	T3s #113	T2s #118
A9 #25	K9 #40	Q9 #55	J9 #66	T9 #73	99 #6	98s #81	97s #93	96s #103	95s #114	94s #123	93s #127	92s #132
A8 #32	K8 #54	Q8 #68	J8 #77	T8 #88	98 #96	88 #7	87s #98	86s #110	85s #119	84s #129	83s #139	82s #141
A7 #36	K7 #58	Q7 #78	J7 #91	T7 #99	97 #108	87 #116	77 #9	76s #115	75s #124	74s #134	73s #144	72s #152
A6 #45	K6 #65	Q6 #83	J6 #102	T6 #111	96 #120	86 #126	76 #133	66 #15	65s #128	64s #138	63s #147	62s #156
A5 #46	K5 #70	Q5 #90	J5 #107	T5 #122	95 #130	85 #136	75 #142	65 #145	55 #24	54s #137	53s #146	52s #154
A4 #49	K4 #76	Q4 #95	J4 #112	T4 #125	94 #140	84 #148	74 #151	64 #155	54 #153	44 #42	43s #150	42s #159
A3 #56	K3 #82	Q3 #101	J3 #117	T3 #131	93 #143	83 #157	73 #160	63 #162	53 #161	43 #164	33 #61	32s #163
A2 #60	K2 #87	Q2 #105	J2 #121	T2 #135	92 #149	82 #158	72 #165	62 #167	52 #166	42 #168	32 #169	22 #80

10 PLAYERS

AA #1	AKs #4	AQs #6	AJs #8	ATs #13	A9s #19	A8s #25	A7s #30	A6s #36	A5s #29	A4s #32	A3s #33	A2s #37
AK #11	KK #2	KQs #7	KJs #10	KTs #14	K9s #22	K8s #38	K7s #45	K6s #53	K5s #55	K4s #58	K3s #59	K2s #60
AQ #18	KQ #20	QQ #3	QJs #12	QTs #15	Q9s #26	Q8s #42	Q7s #63	Q6s #66	Q5s #69	Q4s #70	Q3s #72	Q2s #74
AJ #28	KJ #31	QJ #35	JJ #5	JTs #16	J9s #24	J8s #41	J7s #61	J6s #79	J5s #84	J4s #86	J3s #87	J2s #88
AT #43	KT #47	QT #52	JT #50	TT #9	T9s #23	T8s #39	T7s #57	T6s #75	T5s #93	T4s #96	T3s #97	T2s #99
A9 #77	K9 #81	Q9 #83	J9 #80	T9 #73	99 #17	98s #40	97s #54	96s #68	95s #89	94s #106	93s #107	92s #110
A8 #91	K8 #112	Q8 #116	J8 #111	T8 #100	98 #98	88 #21	87s #51	86s #64	85s #78	84s #94	83s #115	82s #117
A7 #104	K7 #122	Q7 #131	J7 #129	T7 #124	97 #120	87 #113	77 #27	76s #56	75s #67	74s #85	73s #102	72s #119
A6 #114	K6 #125	Q6 #138	J6 #147	T6 #141	96 #136	86 #126	76 #121	66 #34	65s #62	64s #71	63s #90	62s #108
A5 #101	K5 #128	Q5 #140	J5 #149	T5 #157	95 #150	85 #139	75 #130	65 #123	55 #44	54s #65	53s #76	52s #92
A4 #105	K4 #132	Q4 #143	J4 #152	T4 #159	94 #164	84 #156	74 #145	64 #134	54 #127	44 #46	43s #82	42s #95
A3 #109	K3 #133	Q3 #144	J3 #154	T3 #161	93 #165	83 #167	73 #160	63 #148	53 #137	43 #142	33 #48	32s #103
A2 #118	K2 #135	Q2 #146	J2 #155	T2 #162	92 #166	82 #168	72 #169	62 #163	52 #151	42 #153	32 #158	22 #49





Risk®

Basic Risk Mechanic

- Attacker rolls *(up to)* 3 dice
- Defender rolls *(up to)* 2 dice

- Highest dice attacks highest dice
- In a tie, defender wins

Sometimes Brute-Force is easier!

```
For Attack1 = 1 to 6
 For Attack2 = 1 to 6
  For Attack3 = 1 to 6

            AttackHigh = Highest (Attack1, Attack2, Attack3)
            AttackMedium = Medium (Attack1, Attack2, Attack3)

   For Defence1 = 1 to 6
    For Defence2 = 1 to 6

             DefenceHigh = Highest (Defence1, Defence2)
             DefenceLow = Lowest (Defence1, Defence2)
             Calculate_Win_Loss_Tie (AttackHigh, AttackMedium, DefenceHigh, DefenceLow)

    Next
   Next

  Next
 Next
Next
```

There are only 7,776 combinations. It’s easier, simpler, and less error-prone to just brute-force and enumerate all combinations

Basic Dice Results

Attacker Rolls | **Defender Rolls 2 Dice** | **Defender Rolls 1 Die**

3 Attacking vs. 2 Defending		
Attack Wins 2	(2890/7776)	37.17%
Defence Wins 2	(2275/7776)	29.26%
Attack 1 Defence 1	(2611/7776)	33.58%

3 Attacking vs. 1 Defending		
Attack Wins	(855/1296)	65.97%
Defence Wins	(441/1296)	34.03%

2 Attacking vs. 2 Defending		
Attack Wins 2	(295/1296)	22.76%
Defence Wins 2	(581/1296)	44.83%
Attack 1 Defence 1	(420/1296)	32.41%

2 Attacking vs. 1 Defending		
Attack Wins	(125/216)	57.87%
Defence Wins	(91/216)	42.13%

1 Attacking vs. 2 Defending		
Attack Wins	(55/216)	25.46%
Defence Wins	(161/216)	74.54%

1 Attacking vs. 1 Defending		
Attack Wins	(15/36)	41.67%
Defence Wins	(21/36)	58.33%

More dice ...

Attacker Rolls — **Defender Rolls 2 Dice** / **Defender Rolls 1 Die**

3 Attacking vs. 2 Defending		
Attack Wins 2	(2890/7776)	37.17%
Defence Wins 2	(2275/7776)	29.26%
Attack 1 Defence 1	(2611/7776)	33.58%

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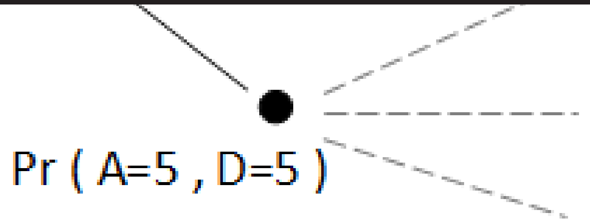
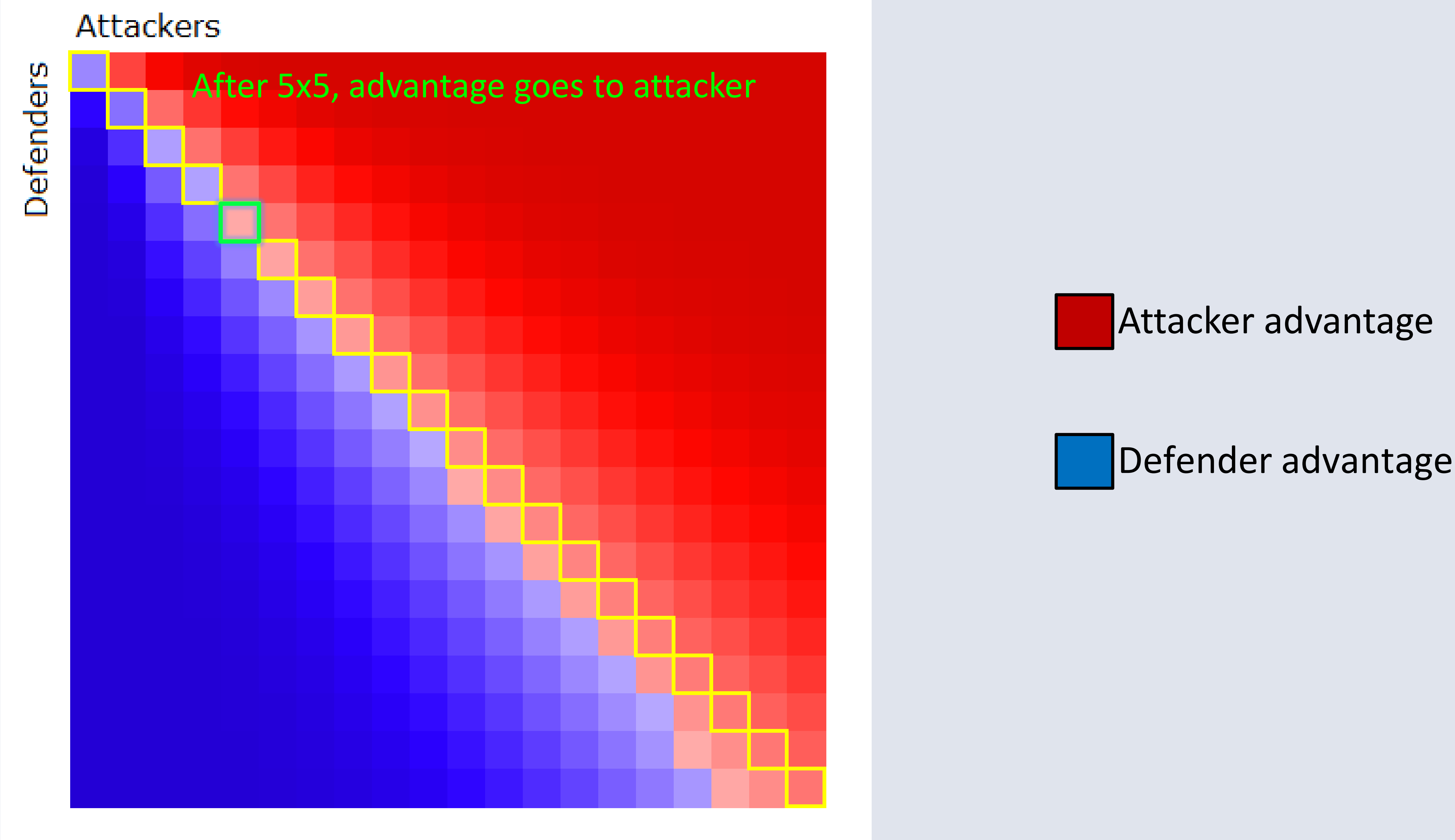


Results

Defenders	Attackers 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	41.667%	75.424%	91.637%	97.154%	99.032%	99.671%	99.888%	99.962%	99.987%	99.996%	99.998%	99.999%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%
2	10.610%	36.265%	65.595%	78.545%	88.979%	93.398%	96.665%	98.031%	99.011%	99.420%	99.709%	99.830%	99.915%	99.950%	99.975%	99.985%	99.993%	99.996%	99.998%	99.999%
3	02.702%	20.607%	47.025%	64.162%	76.937%	85.692%	90.994%	94.680%	96.699%	98.110%	98.839%	99.349%	99.603%	99.781%	99.867%	99.928%	99.956%	99.976%	99.986%	99.992%
4	00.688%	09.130%	31.499%	47.653%	63.829%	74.487%	83.374%	88.780%	92.982%	95.393%	97.204%	98.199%	98.932%	99.321%	99.605%	99.751%	99.857%	99.911%	99.950%	99.969%
5	00.175%	04.913%	20.594%	35.861%	50.620%	63.772%	73.640%	81.841%	87.294%	91.628%	94.304%	96.370%	97.581%	98.498%	99.015%	99.401%	99.612%	99.768%	99.851%	99.912%
6	00.045%	02.135%	13.370%	25.250%	39.675%	52.068%	64.007%	72.956%	80.764%	86.109%	90.522%	93.354%	95.611%	96.991%	98.065%	98.697%	99.180%	99.455%	99.663%	99.779%
7	00.011%	01.133%	08.374%	18.149%	29.742%	42.333%	53.553%	64.294%	72.608%	79.983%	85.205%	89.612%	92.541%	94.929%	96.441%	97.644%	98.377%	98.949%	99.287%	99.547%
8	00.003%	00.490%	05.350%	12.340%	22.405%	32.948%	44.558%	54.736%	64.641%	72.397%	79.412%	84.486%	88.857%	91.838%	94.318%	95.933%	97.242%	98.063%	98.715%	99.112%
9	00.001%	00.259%	03.277%	08.617%	16.156%	25.777%	35.693%	46.399%	55.807%	65.006%	72.303%	78.988%	83.916%	88.227%	91.231%	93.772%	95.466%	96.861%	97.758%	98.482%
10	00.000%	00.112%	02.075%	05.719%	11.828%	19.343%	28.676%	37.987%	47.994%	56.759%	65.383%	72.284%	78.676%	83.457%	87.696%	90.704%	93.284%	95.038%	96.504%	97.465%
11	00.000%	00.059%	01.255%	03.917%	08.292%	14.698%	22.187%	31.173%	39.987%	49.395%	57.629%	65.762%	72.319%	78.447%	83.088%	87.248%	90.246%	92.848%	94.647%	96.169%
12	00.000%	00.025%	00.791%	02.555%	05.942%	10.721%	17.331%	24.704%	33.375%	41.749%	50.650%	58.430%	66.140%	72.395%	78.284%	82.790%	86.869%	89.845%	92.457%	94.290%
13	00.000%	00.013%	00.475%	01.725%	04.079%	07.963%	13.039%	19.735%	26.971%	35.338%	43.328%	51.787%	59.174%	66.515%	72.501%	78.172%	82.551%	86.547%	89.496%	92.107%
14	00.000%	00.006%	00.299%	01.111%	02.875%	05.679%	09.956%	15.221%	21.943%	29.026%	37.110%	44.756%	52.827%	59.869%	66.884%	72.629%	78.102%	82.359%	86.273%	89.189%
15	00.000%	00.003%	00.179%	00.742%	01.941%	04.142%	07.321%	11.889%	17.277%	23.980%	30.904%	38.723%	46.062%	53.788%	60.524%	67.248%	72.775%	78.065%	82.208%	86.040%
16	00.000%	00.001%	00.112%	00.473%	01.351%	02.901%	05.486%	08.964%	13.753%	19.211%	25.868%	32.631%	40.204%	47.264%	54.681%	61.144%	67.605%	72.934%	78.055%	82.089%
17	00.000%	00.001%	00.067%	00.314%	00.900%	02.085%	03.958%	06.871%	10.589%	15.540%	21.034%	27.624%	34.230%	41.572%	48.379%	55.516%	61.732%	67.956%	73.103%	78.068%
18	00.000%	00.000%	00.042%	00.198%	00.620%	01.438%	02.920%	05.081%	08.273%	12.182%	17.252%	22.755%	29.265%	35.717%	42.844%	49.419%	56.302%	62.293%	68.300%	73.280%
19	00.000%	00.000%	00.025%	00.131%	00.408%	01.021%	02.073%	03.833%	06.248%	09.675%	13.736%	18.890%	24.381%	30.804%	37.106%	44.031%	50.393%	57.043%	62.829%	68.637%
20	00.000%	00.000%	00.016%	00.082%	00.279%	00.696%	01.510%	02.788%	04.803%	07.441%	11.066%	15.246%	20.457%	25.922%	32.251%	38.410%	45.145%	51.310%	57.746%	63.343%

A picture paints a thousand numbers

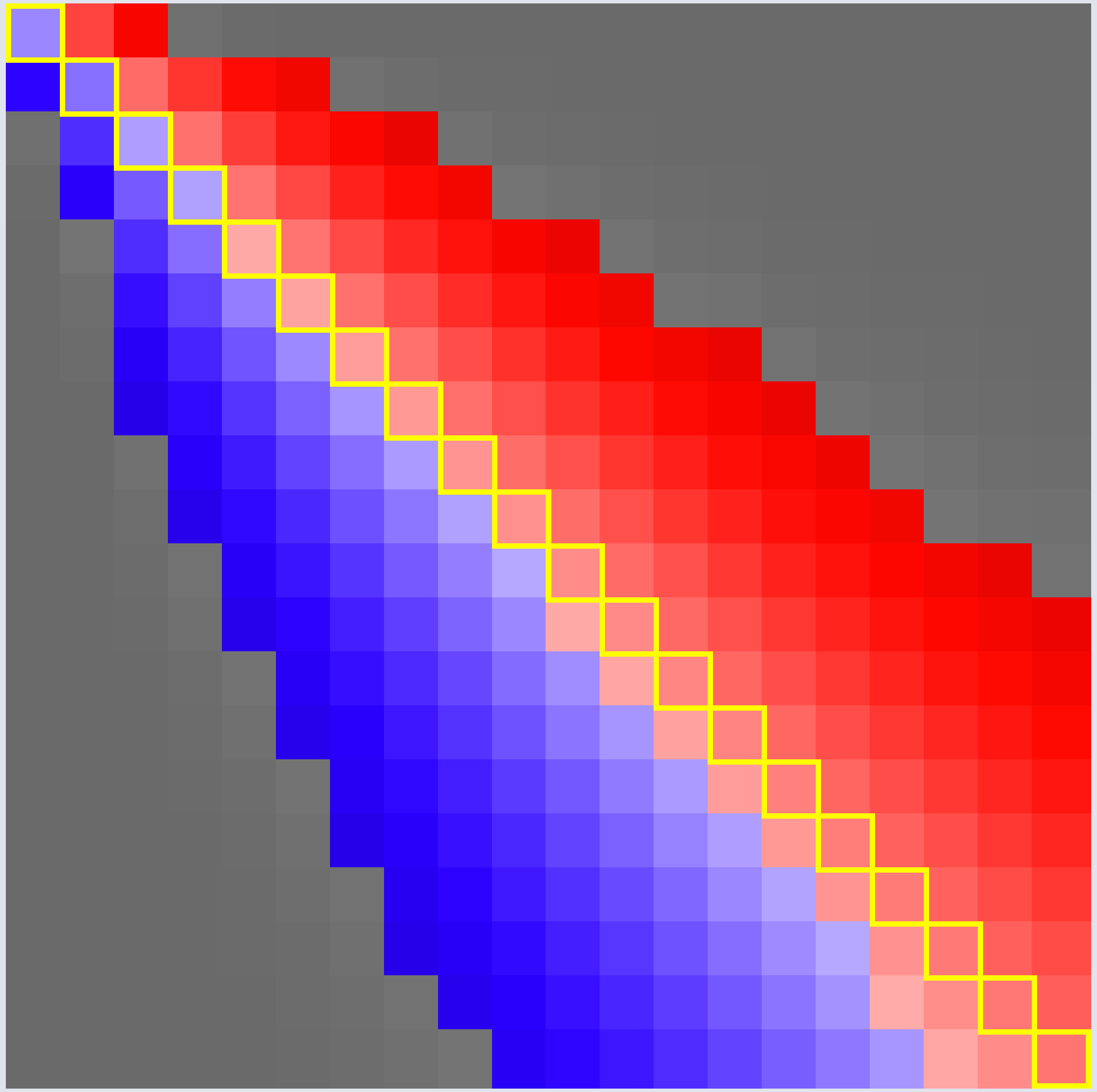


Results

STRATEGY TIP – It's better to attack then defend. Be aggressive.

STRATEGY TIP – Always attack with superior numbers to maximize the chances of your attack being successful.

STRATEGY TIP – If attacking a region with the same number of armies as the defender, make sure that you have *at least* five armies if you want the odds in your favour (the more the better).



95% confidence level

Yahtzee

What is the probability of rolling a Yahtzee?

In one roll, it's 1/6 x 1/6 x 1/6 x 1/6 = 1/1296

But what about over three rolls?

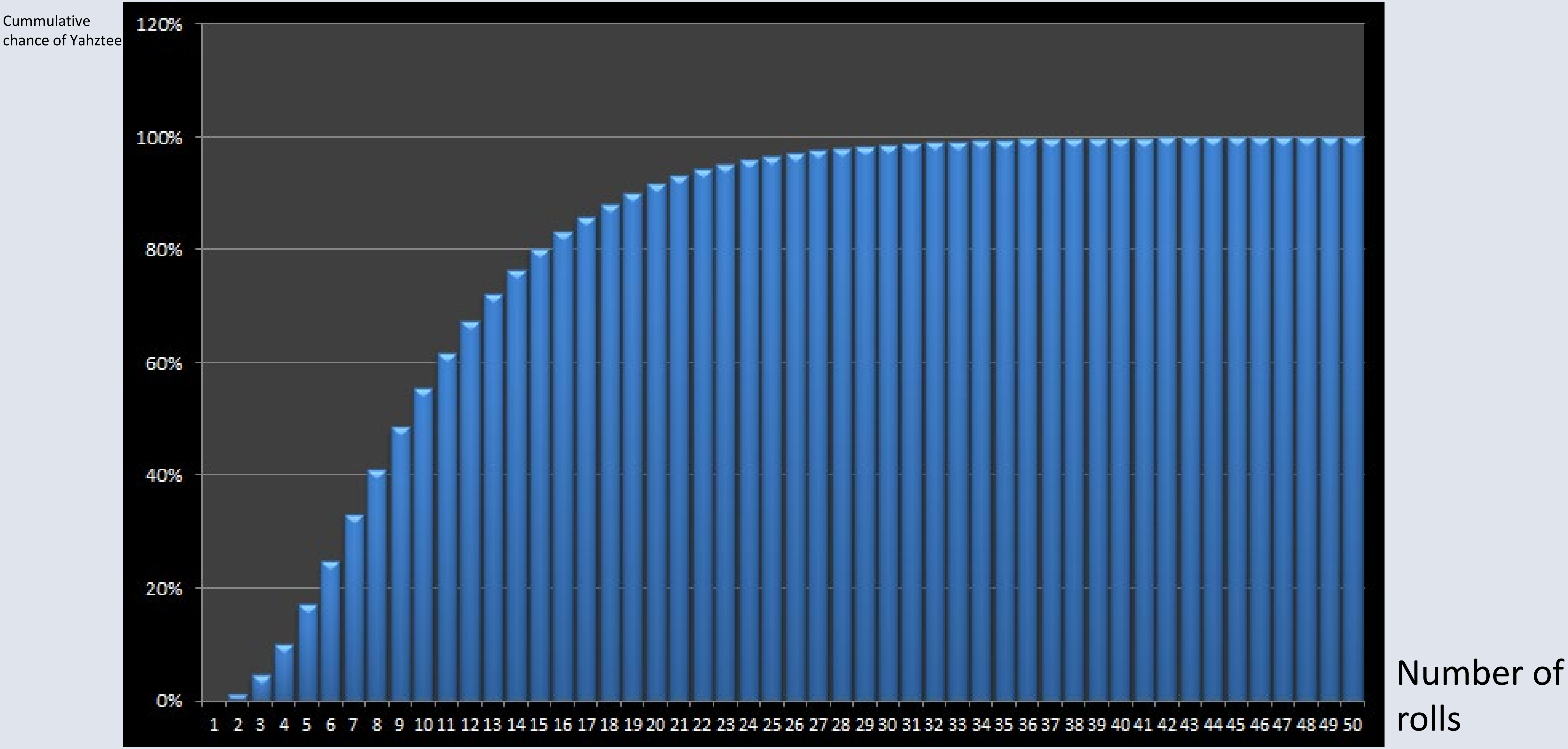
$$
\begin{array}{c|c}
 & \begin{matrix} 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 \end{matrix} \\
\begin{matrix} 1 \\[6pt] 2 \\[6pt] 3 \\[6pt] 4 \\[6pt] 5 \end{matrix} &
\begin{pmatrix}
\frac{120}{1296} & \frac{900}{1296} & \frac{250}{1296} & \frac{25}{1296} & \frac{1}{1296} \\
 & \frac{120}{216} & \frac{80}{216} & \frac{15}{216} & \frac{1}{216} \\
 & & \frac{25}{36} & \frac{10}{36} & \frac{1}{36} \\
 & & & \frac{5}{6} & \frac{1}{6} \\
 & & & & 1
\end{pmatrix}
\end{array}
$$

Markov Chain – Transition Matrix

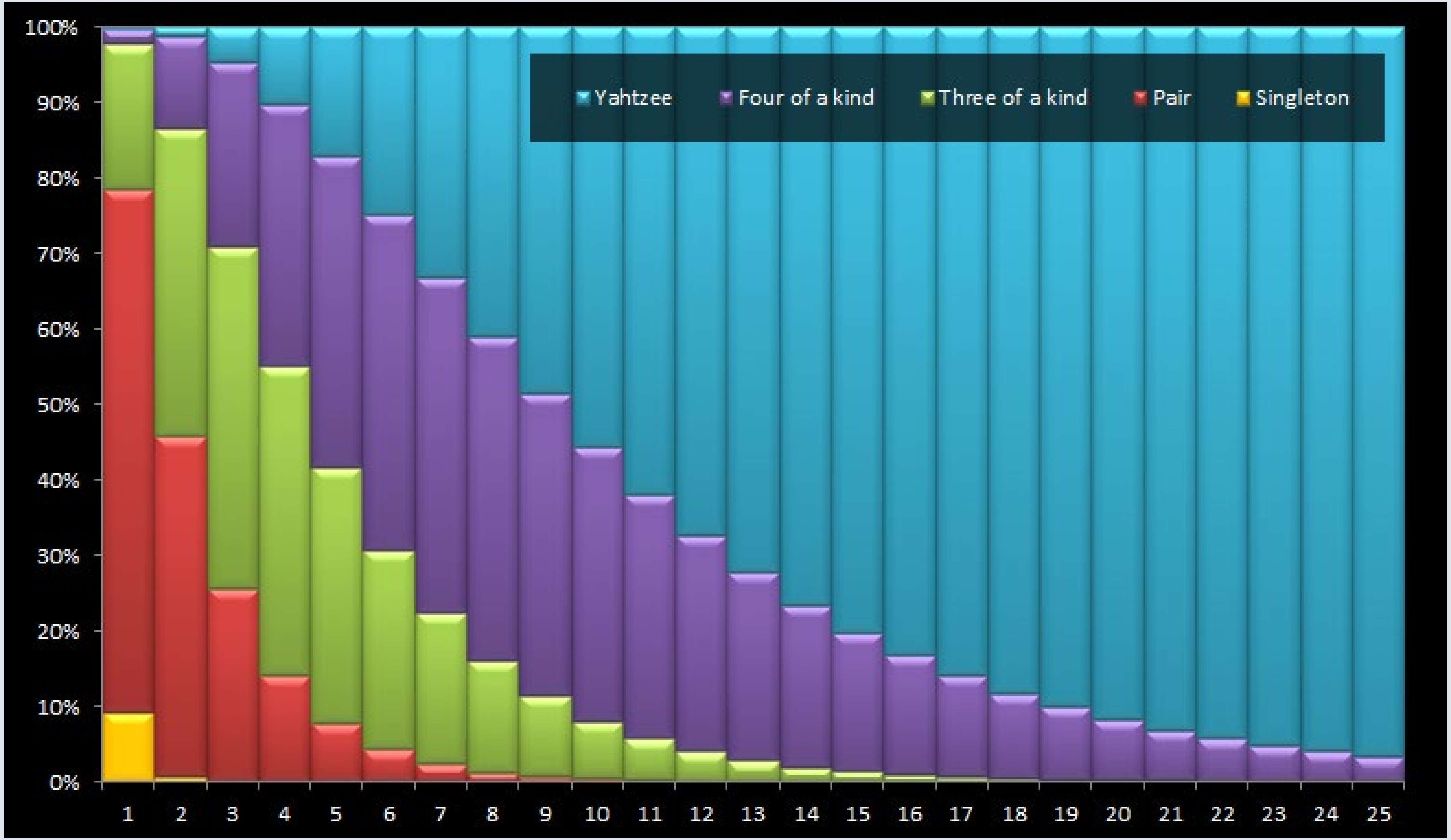
Watch out! Here you may elect to change your target!

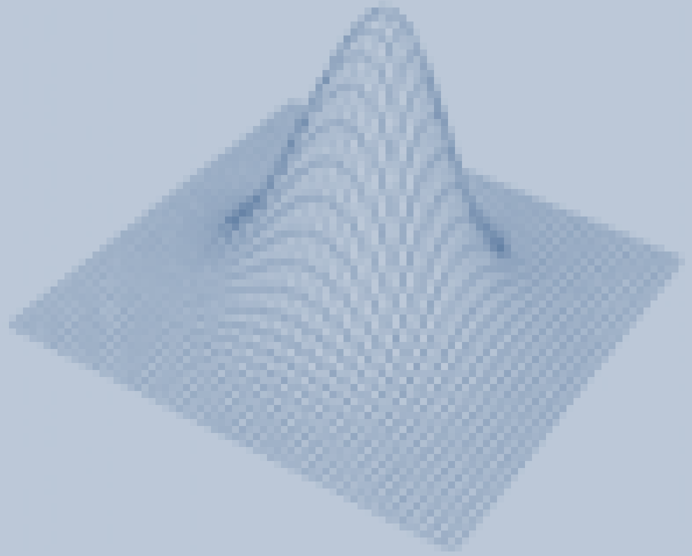
Answer = 4.6029%

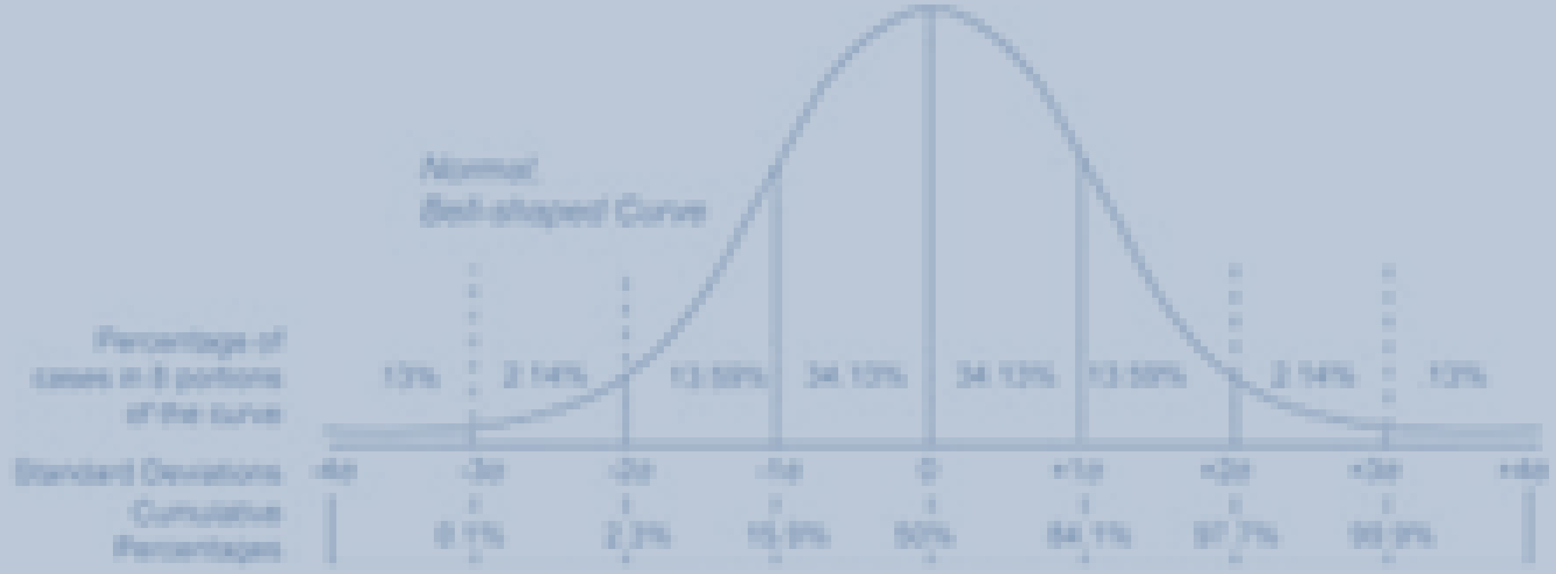
Yahztee - “Just one more roll?”



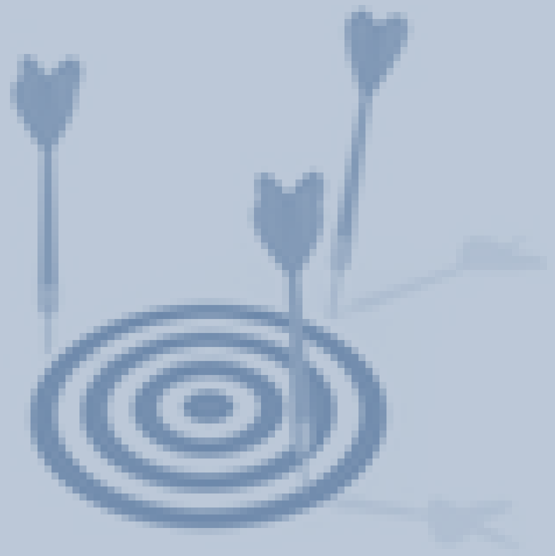
Breakdown of odds

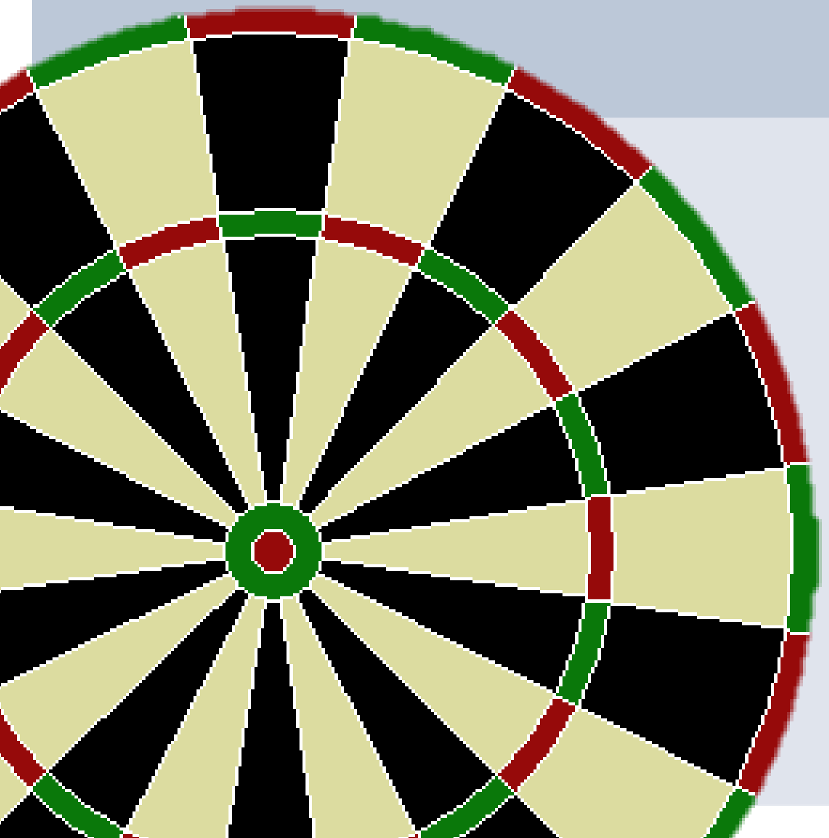






$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

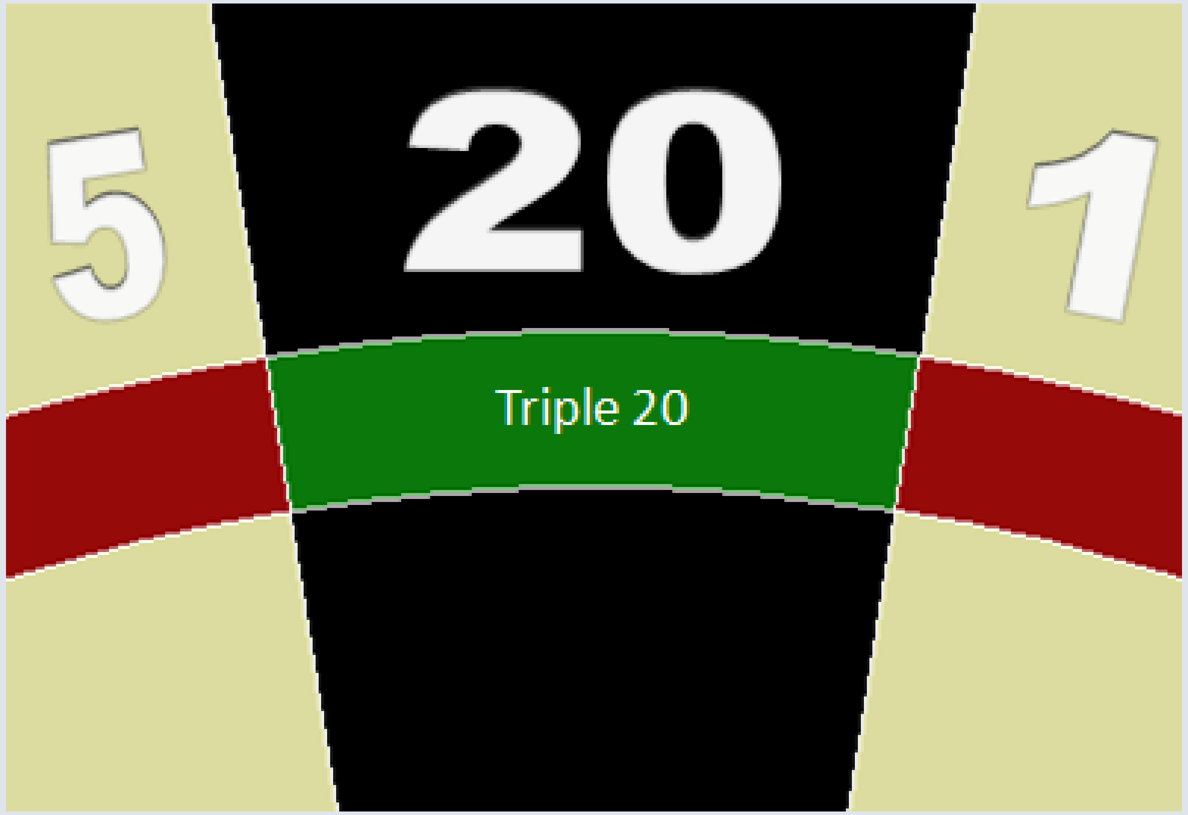




Darts

Where is the best place to aim on a dartboard?

(To maximize expected score)

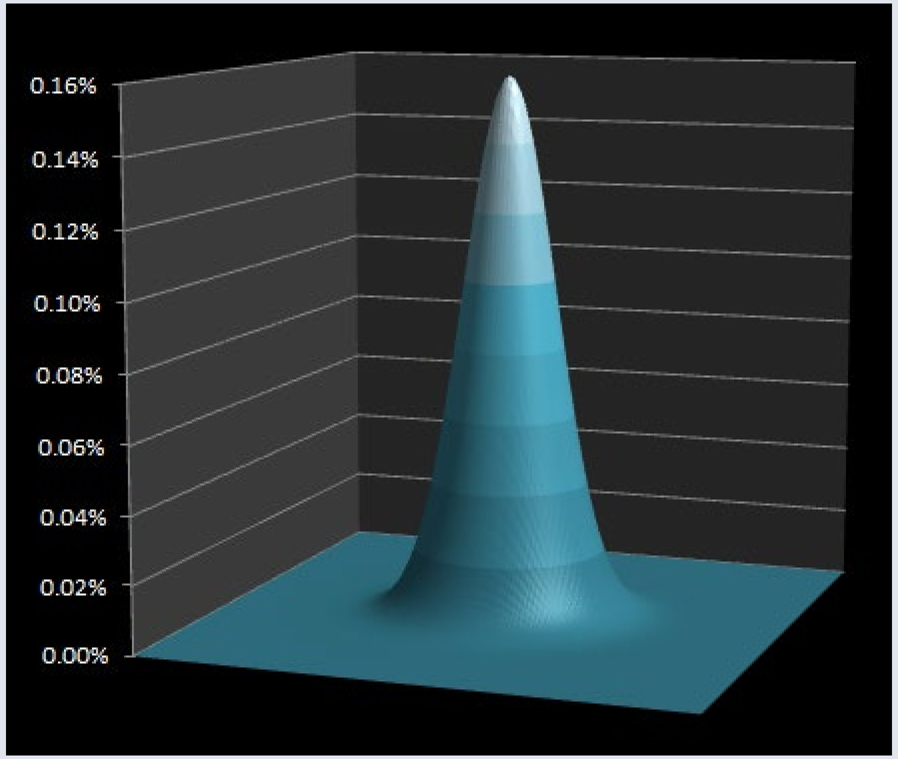


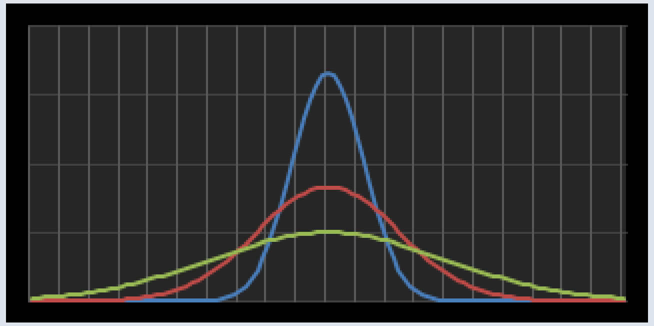
It depends on how good you are!


20
1
18
4
13
6
10
15
2
17
3
19
7
16
8
11
14
9
12
5

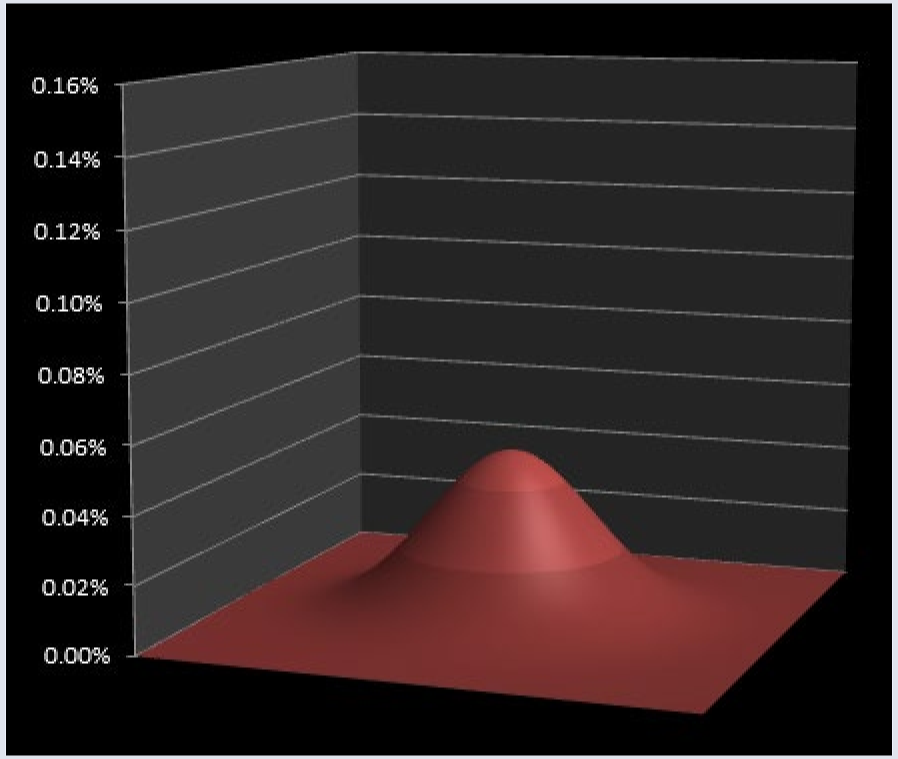
“Good” Players

“Bad” Players



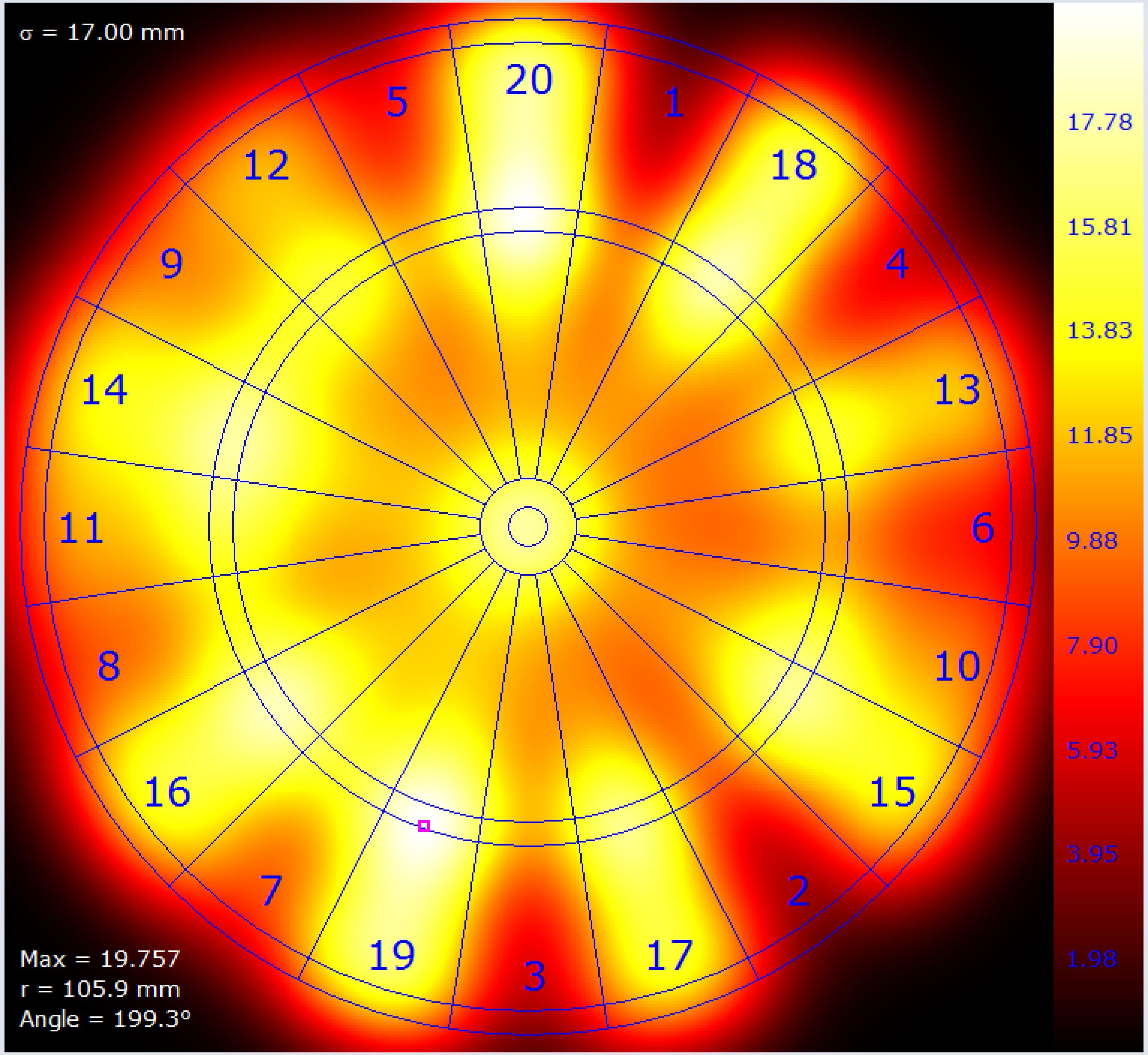


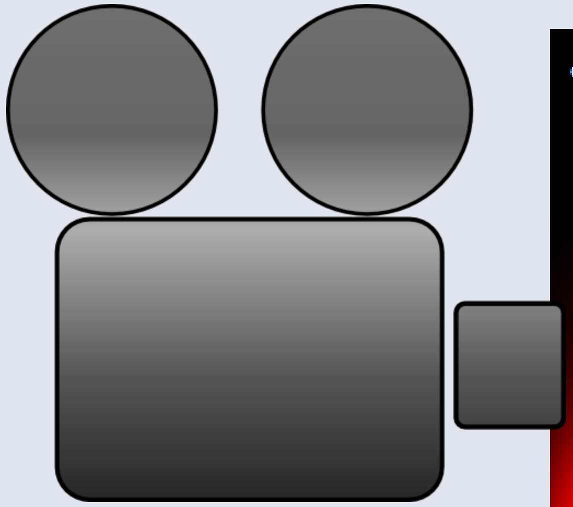
σ

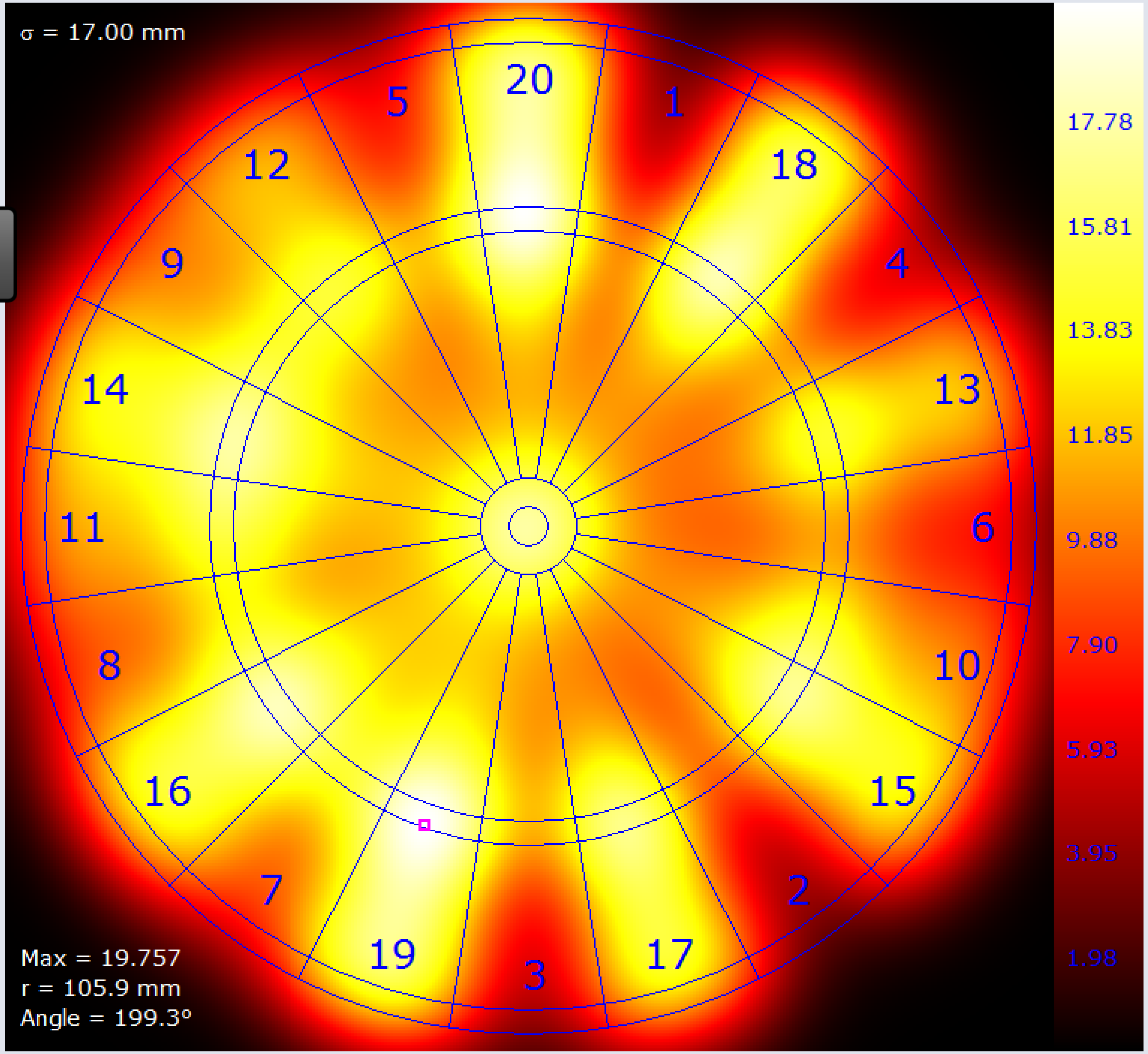


High accuracy
Low standard deviation

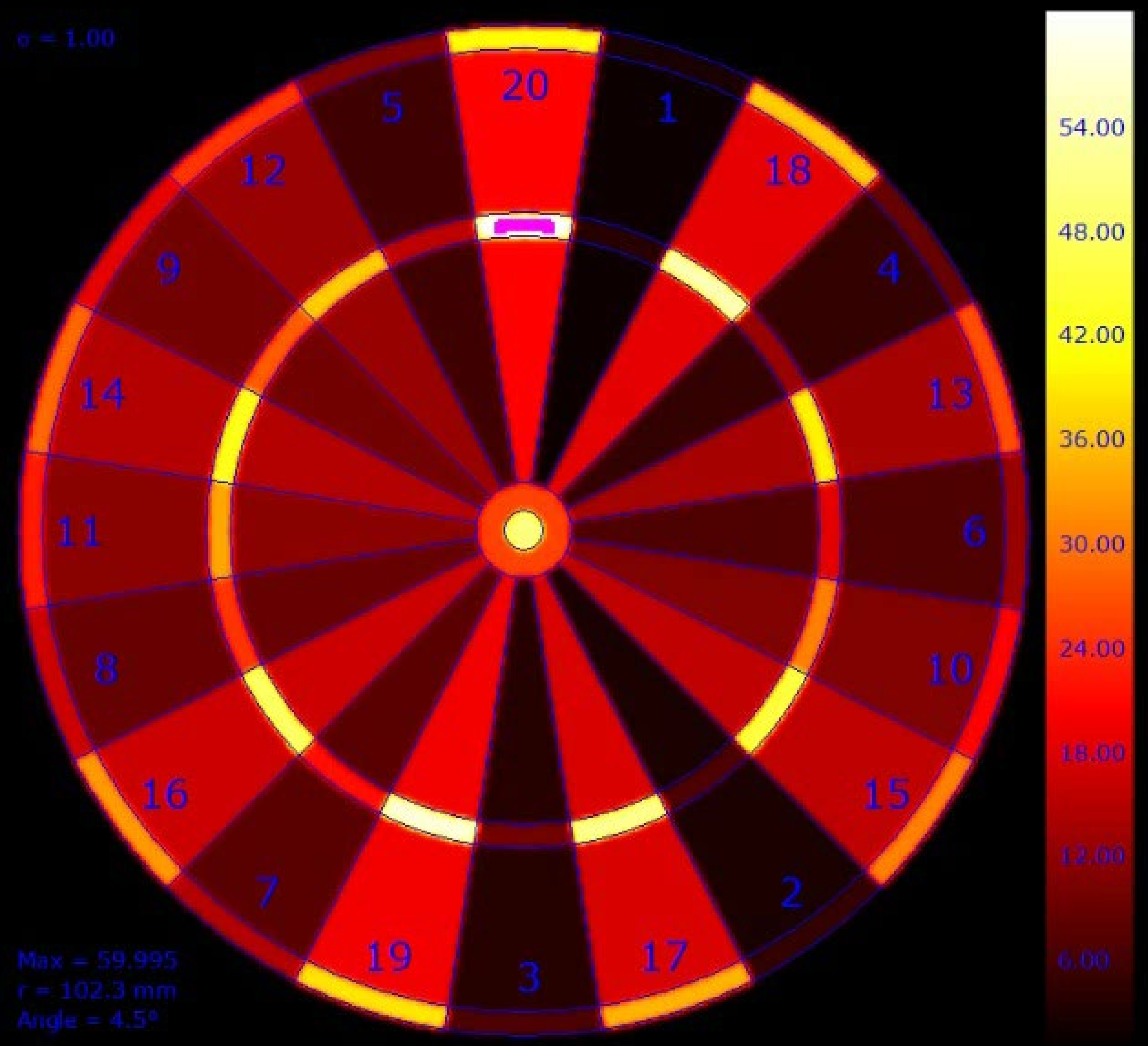
Low accuracy
High standard deviation

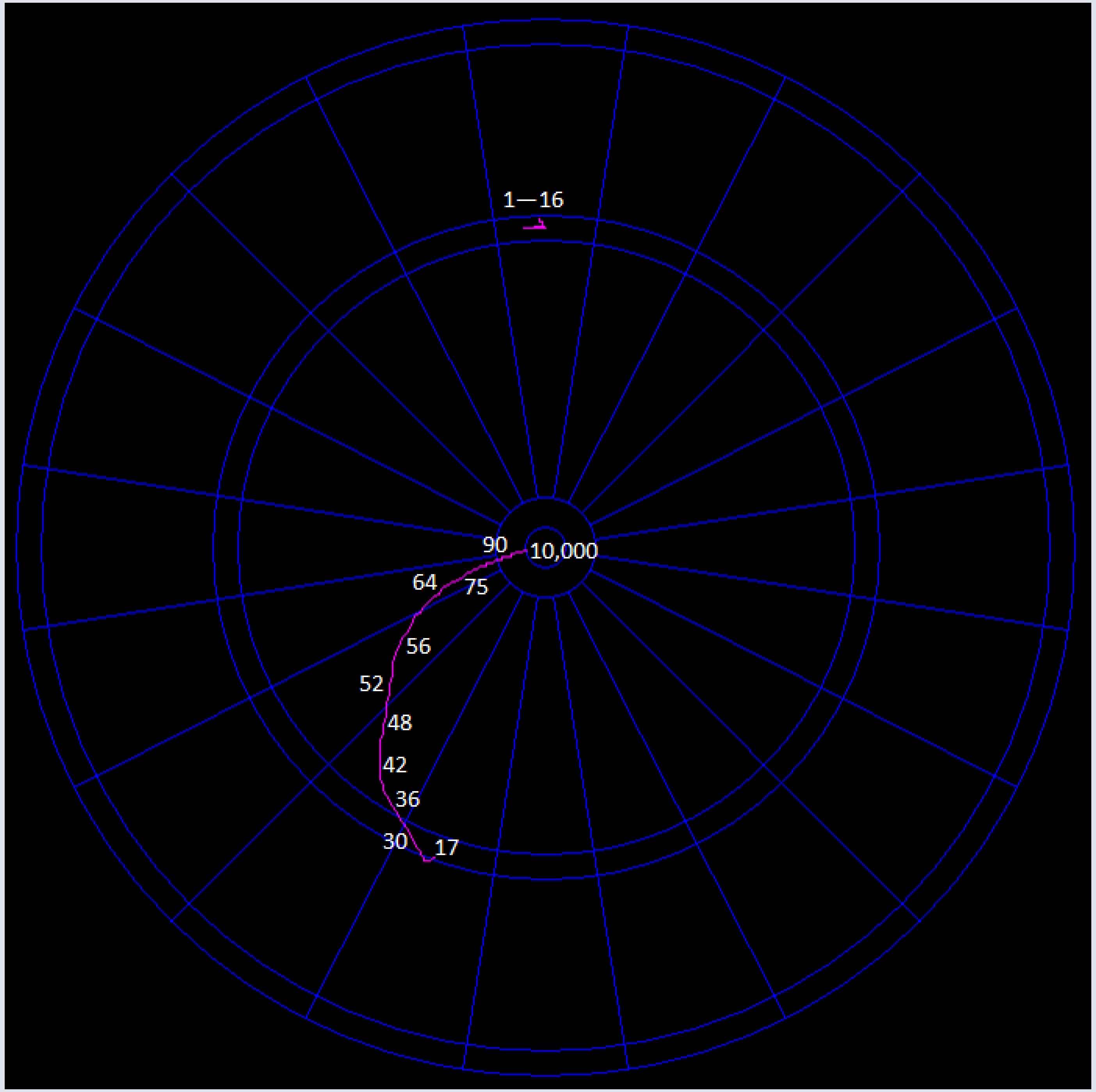

σ = 17.00 mm
20
1
18
4
13
6
10
15
2
17
3
19
7
16
8
11
14
9
12
5
17.78
15.81
13.83
11.85
9.88
7.90
5.93
3.95
1.98
Max = 19.757
r = 105.9 mm
Angle = 199.3°

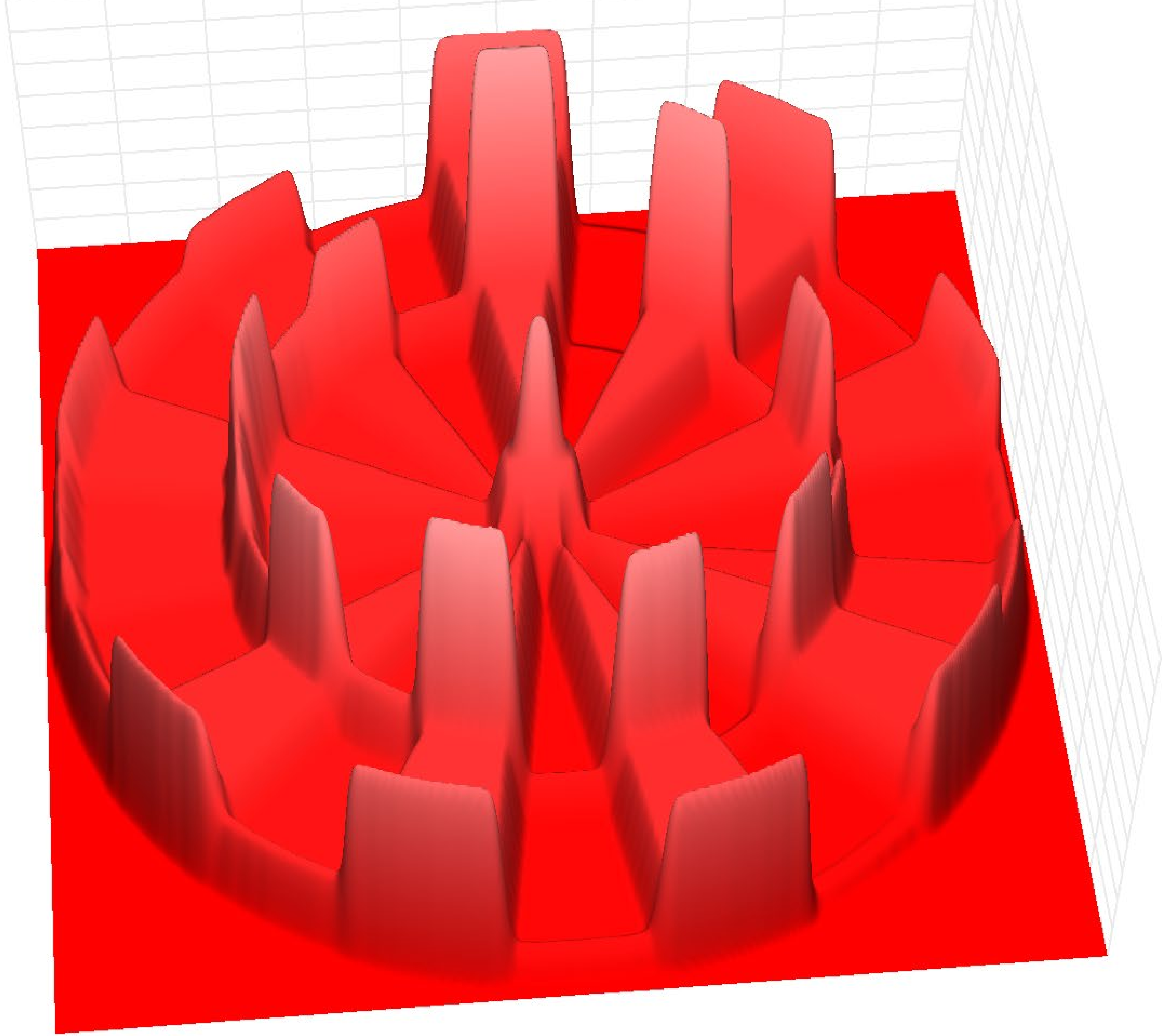


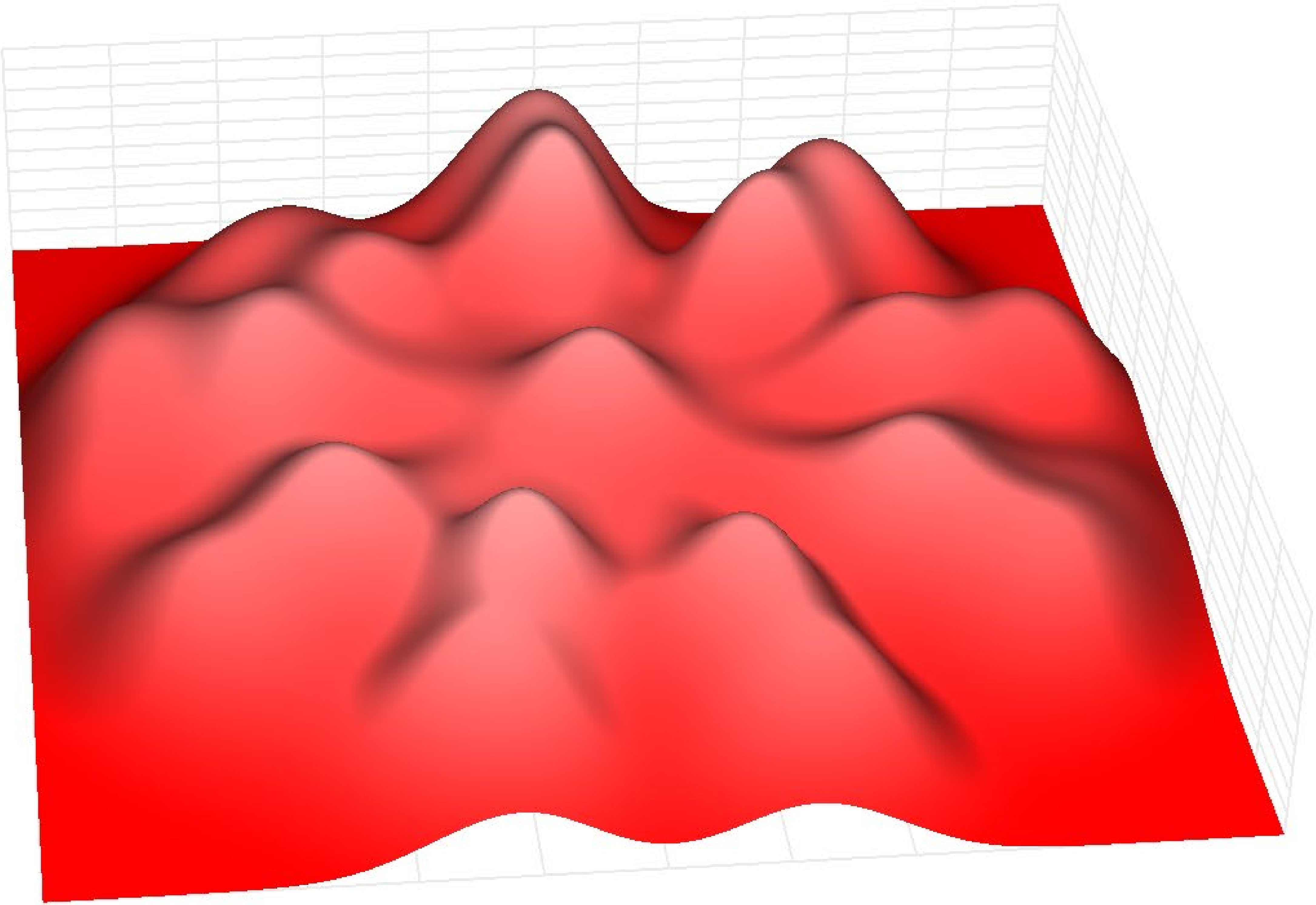

σ = 17.00 mm
20
1
18
4
13
6
10
15
2
17
3
19
7
16
8
11
14
9
12
5
Max = 19.757
r = 105.9 mm
Angle = 199.3°
17.78
15.81
13.83
11.85
9.88
7.90
5.93
3.95
1.98

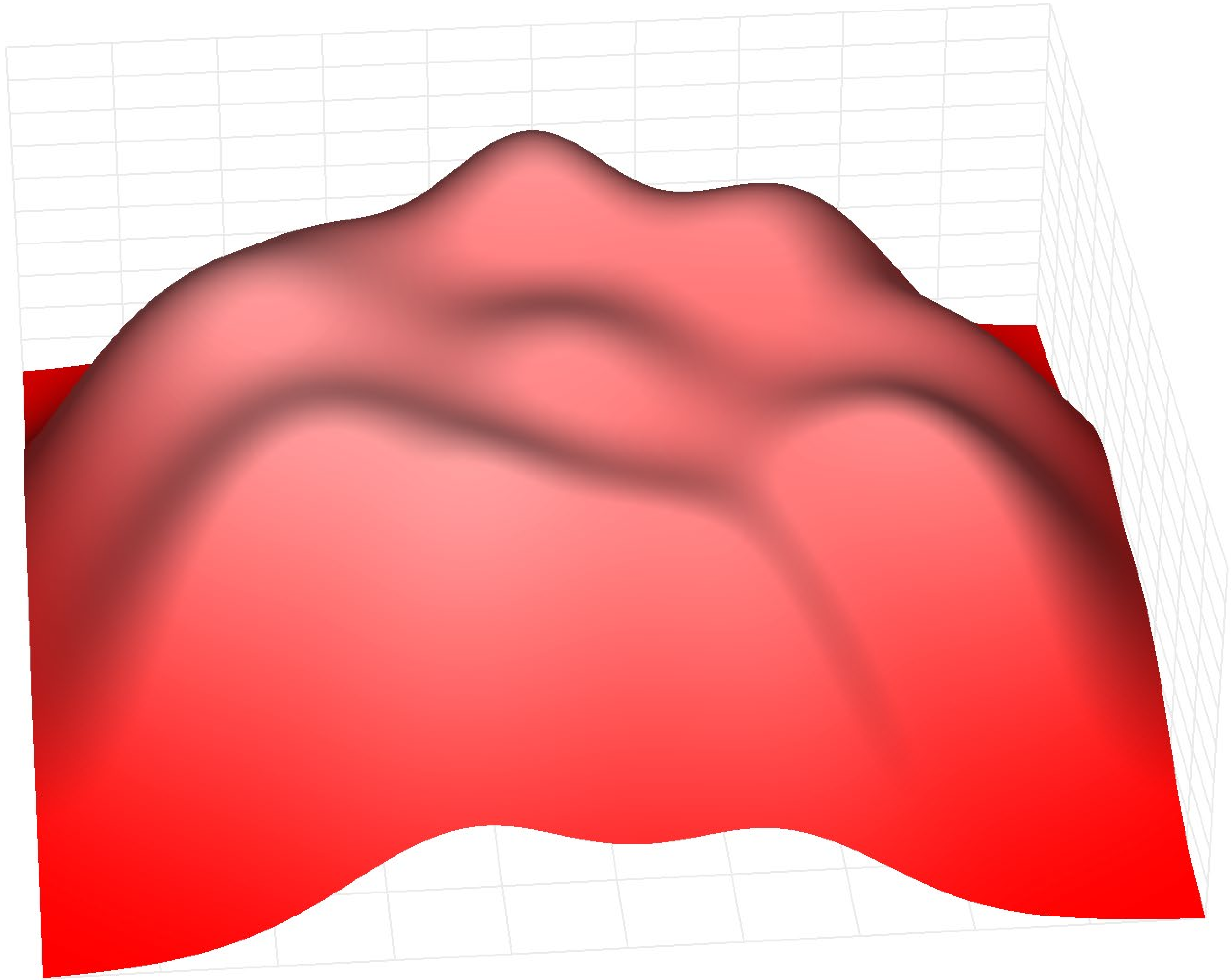
Animation


σ = 1.00
20
1
18
4
13
6
10
15
2
17
3
19
7
16
8
11
14
9
12
5
Max = 59.995
r = 102.3 mm
Angle = 4.5°
54.00
48.00
42.00
36.00
30.00
24.00
18.00
12.00
6.00

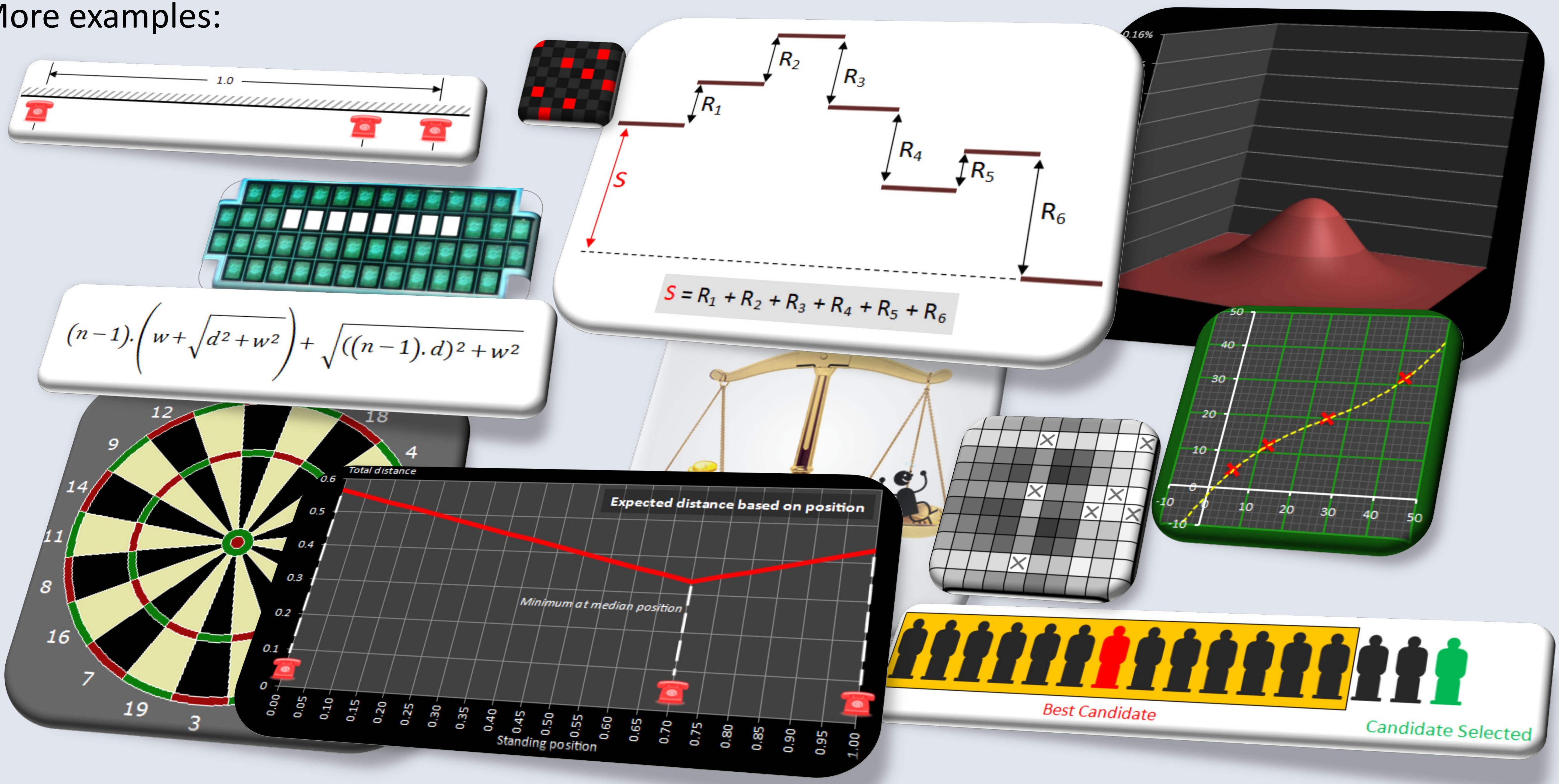

1—16
90
10,000
64
75
56
52
48
42
36
30
17





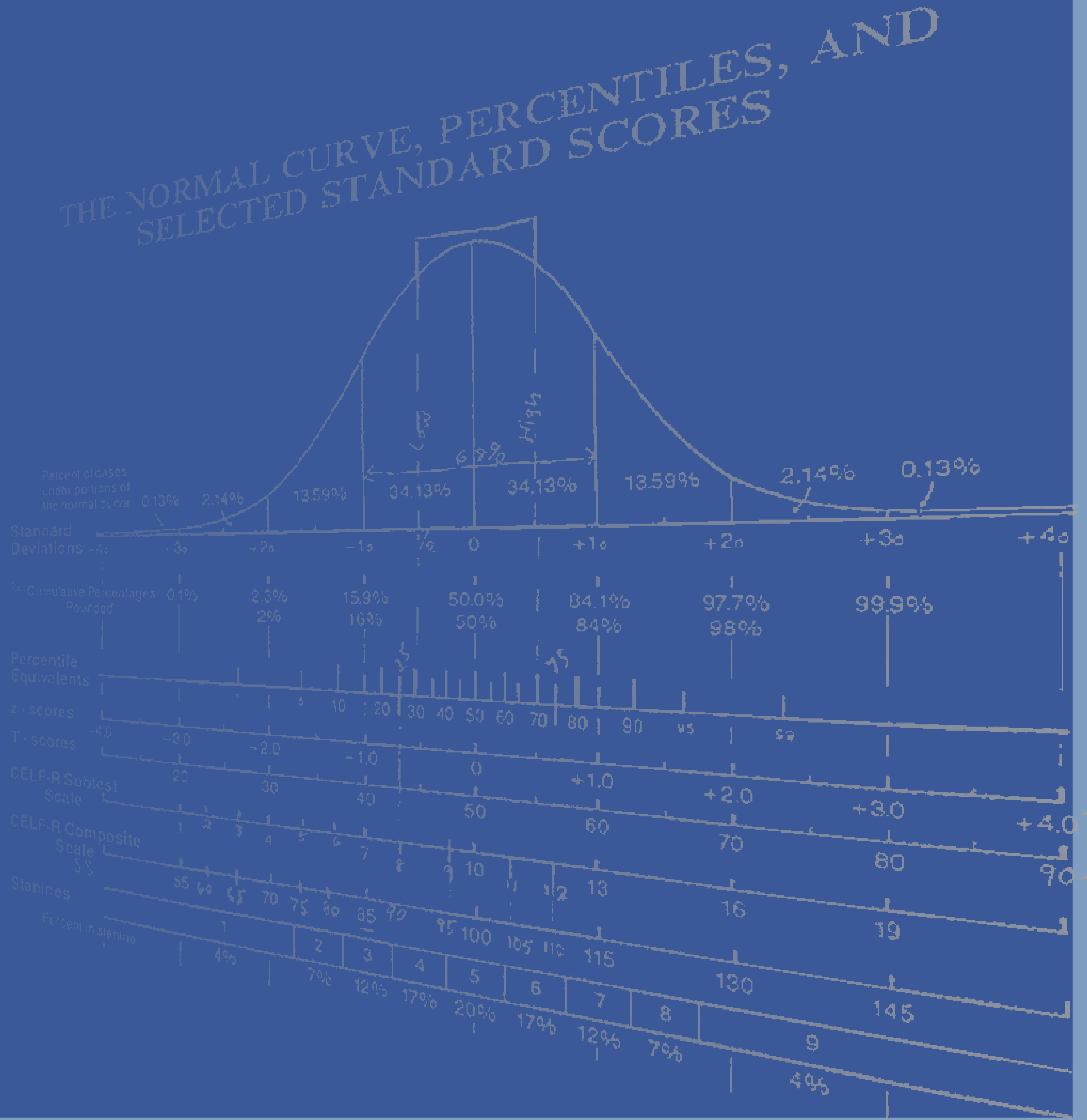


More examples:



http://DataGenetics.com/blog

THE END


THE NORMAL CURVE, PERCENTILES, AND SELECTED STANDARD SCORES
Standard Deviations
Percentile Equivalents
z-scores
T-scores
CELF-R Subtest Scale
CELF-R Composite Scale
Stanines