# Fluids in Games

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## **Introductory Bits**

- General summary with some details
- Not a fluids expert
- Theory and examples

- Deformable
- Flowing
- Examples
  - Smoke
  - Fire
  - Water







Fluids have variable density

 (Density field)

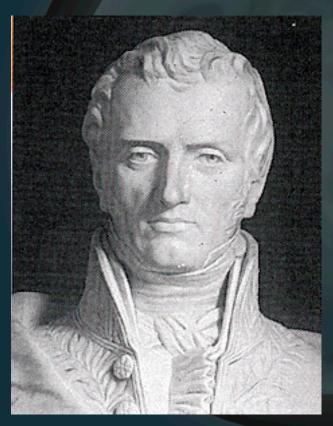
Fluids "flow"
 – (Vector field)

Need way to represent

 Density (x)
 Velocity (u)
 Sometimes temperature

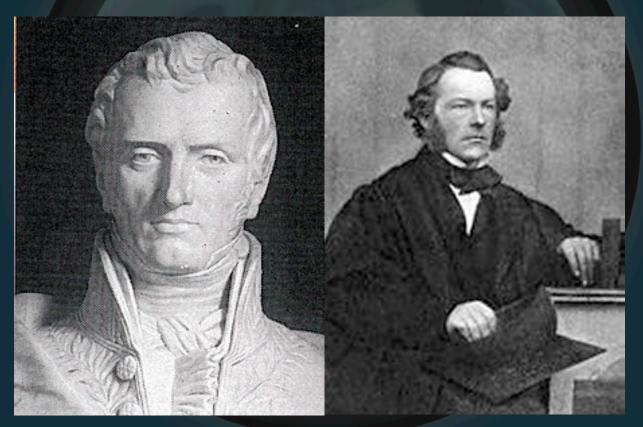
• Our heroes:

### • Our heroes:



#### Navier

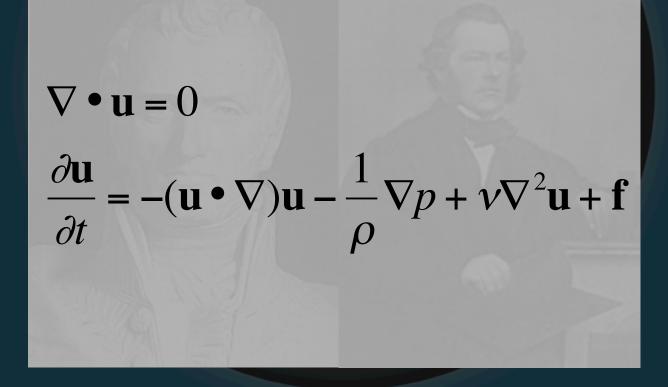
#### • Our heroes:



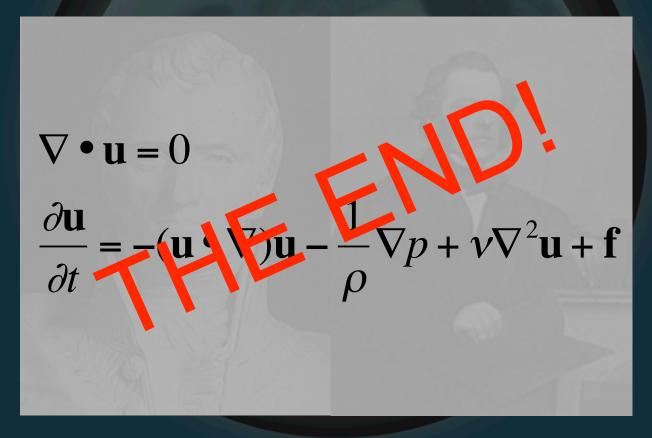
#### Navier



• Their creation:

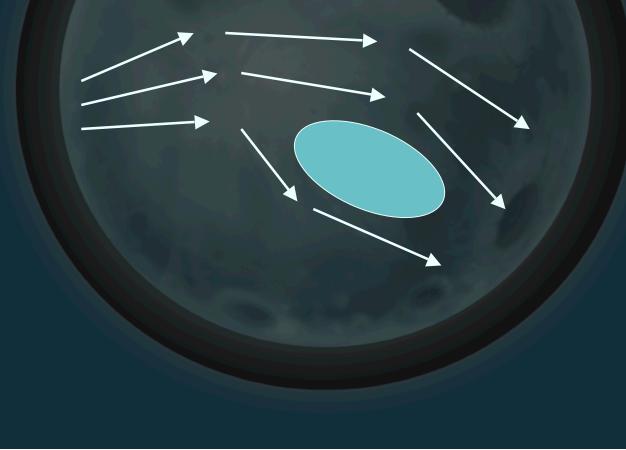


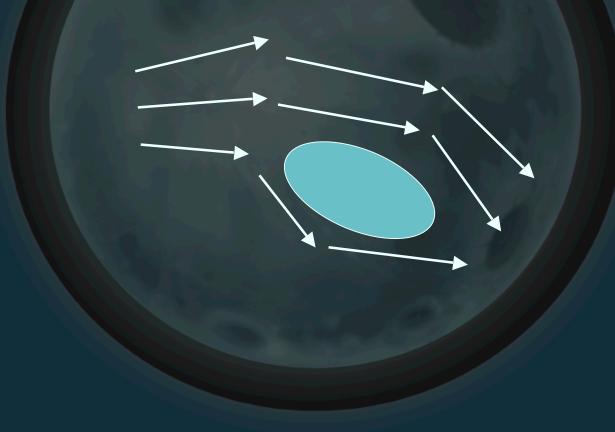
• Their creation:

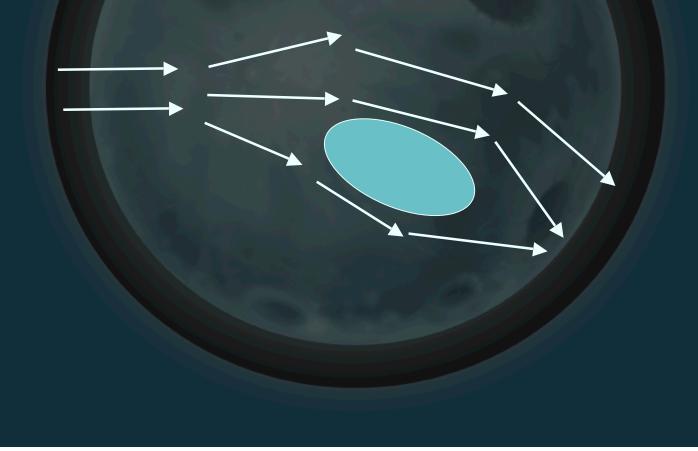


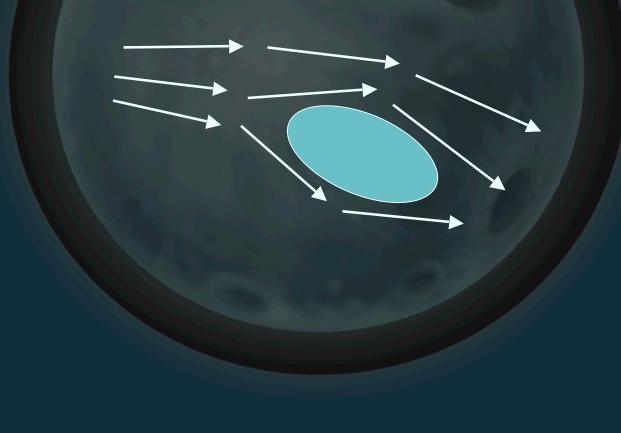
• Their creation:

 $\nabla \cdot \mathbf{u} = 0$  $\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu\nabla^{2}\mathbf{u} + \mathbf{f}$ SERIOUSLY ---WHAT DOES THIS MEAN?









• What affects it?

What affects it?

#### **External Forces**

Viscosity

• What affects it?

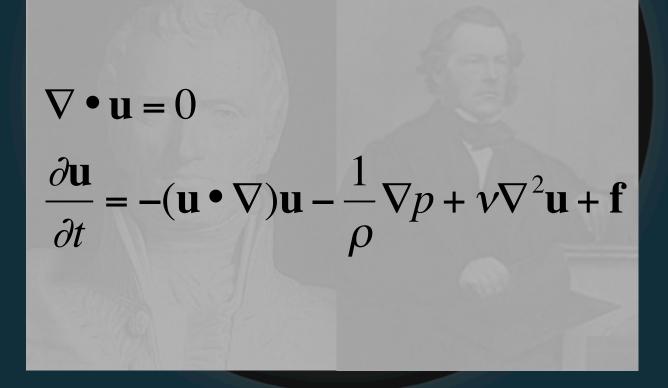
Advection

• What affects it?

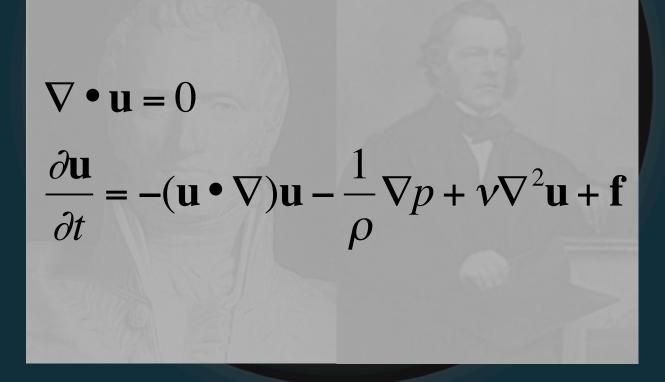
What affects it?

Pressure

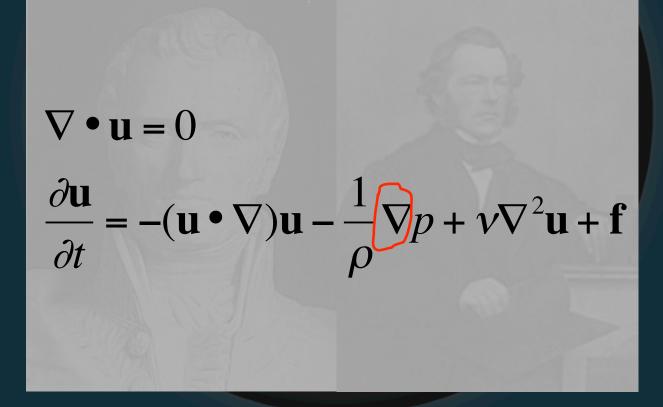
Back to Navier-Stokes



#### Brief notational diversion

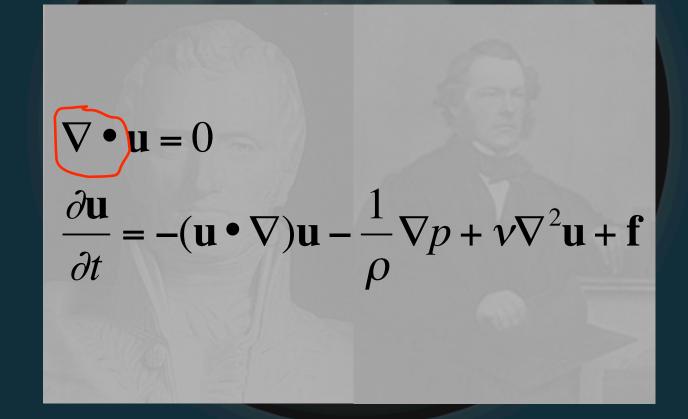


#### Brief notational diversion



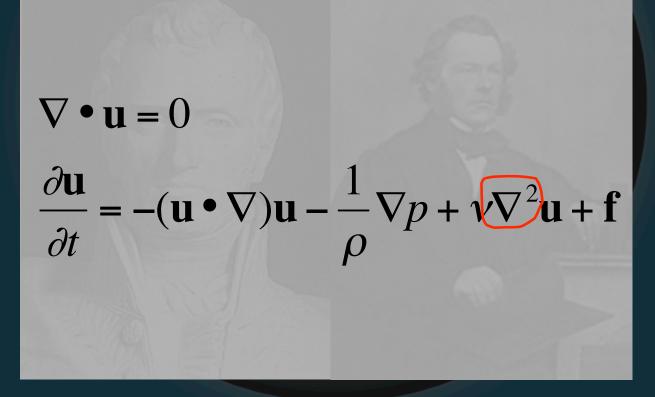
#### Gradient (vector along partial derivative)

#### Brief notational diversion



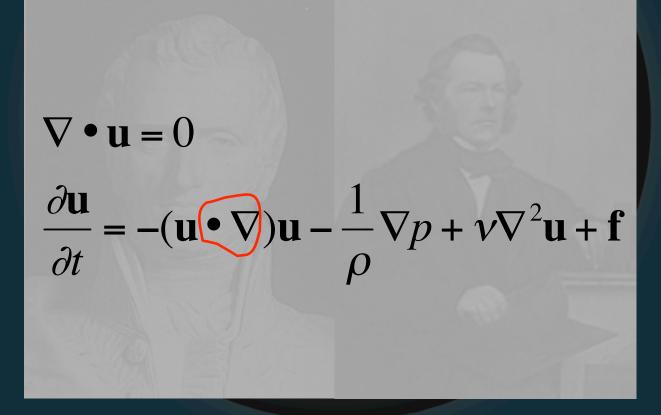
#### Divergence (real derivative of vec. field)

#### Brief notational diversion



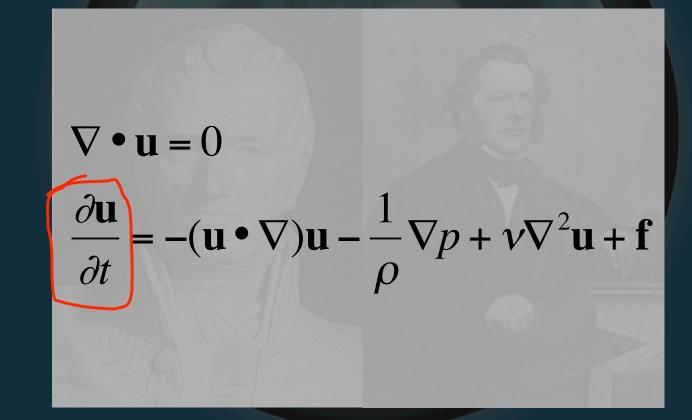
#### Laplacian (divergence of gradient)

#### Brief notational diversion



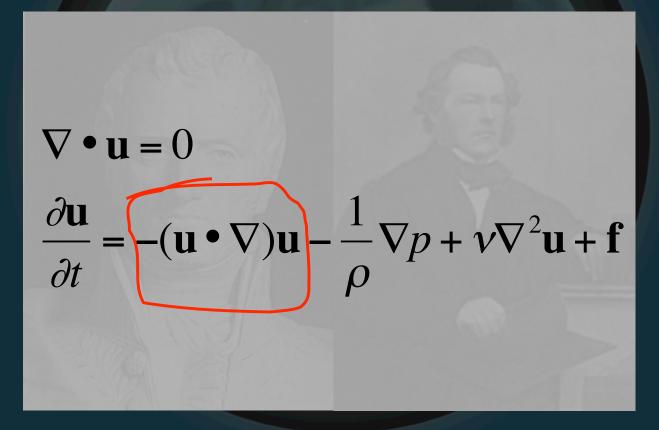
Advection operator (transport of flow)

Back to Navier-Stokes



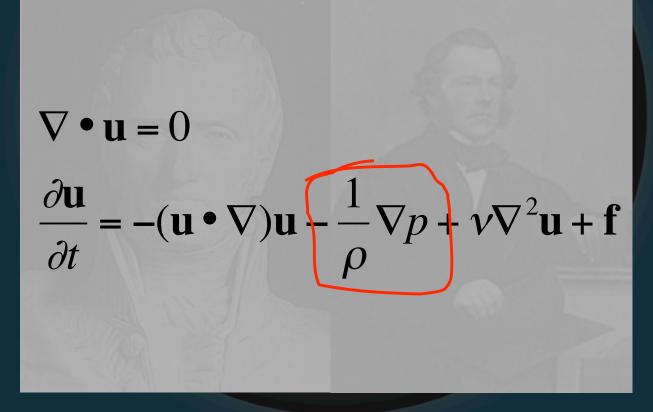
Change in Velocity

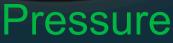
Back to Navier-Stokes



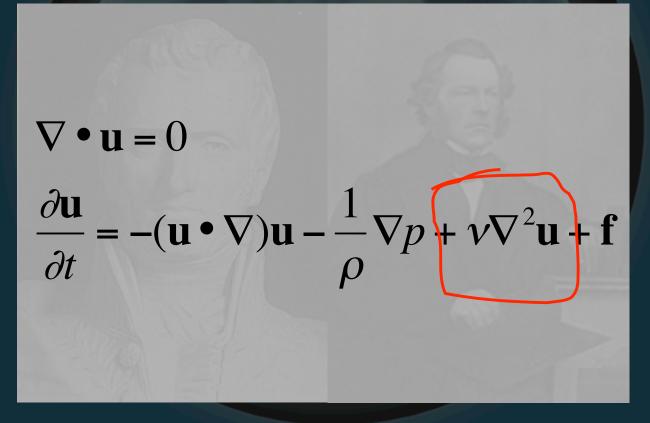
#### Advection

Back to Navier-Stokes



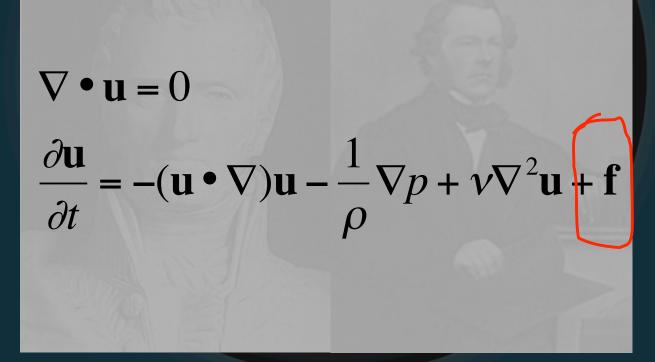


Back to Navier-Stokes



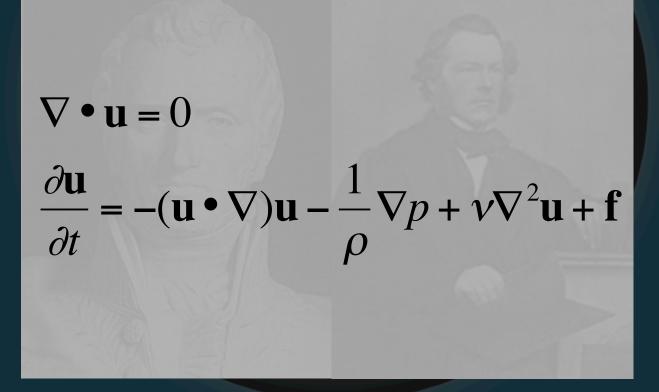
Viscosity

Back to Navier-Stokes



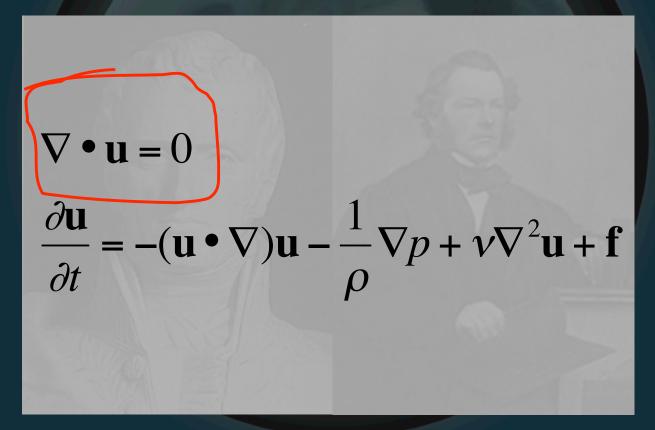
#### **External Forces**

Back to Navier-Stokes



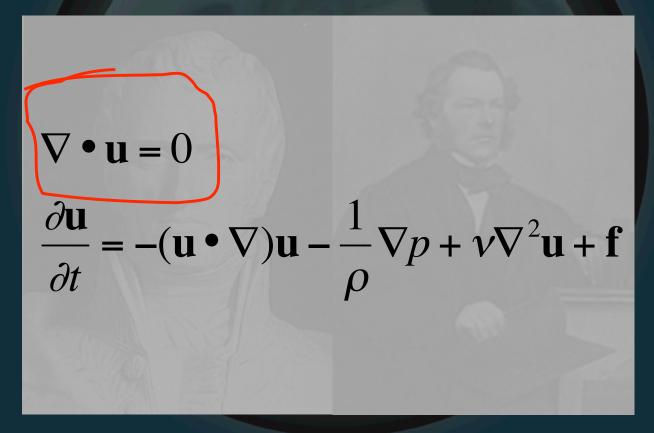
#### HOLD ON THERE BUCKO...

Back to Navier-Stokes



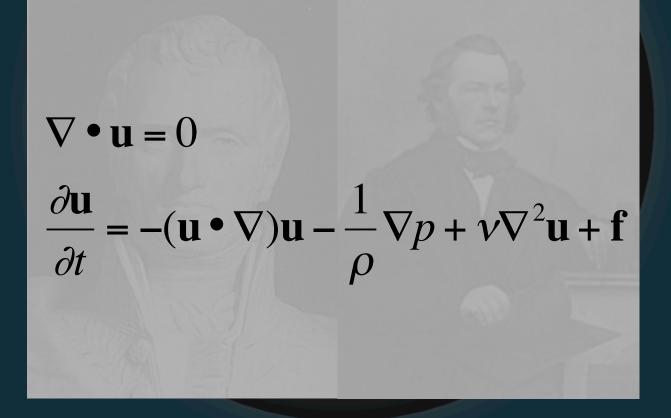
#### WHAT'S THIS ONE?

Back to Navier-Stokes

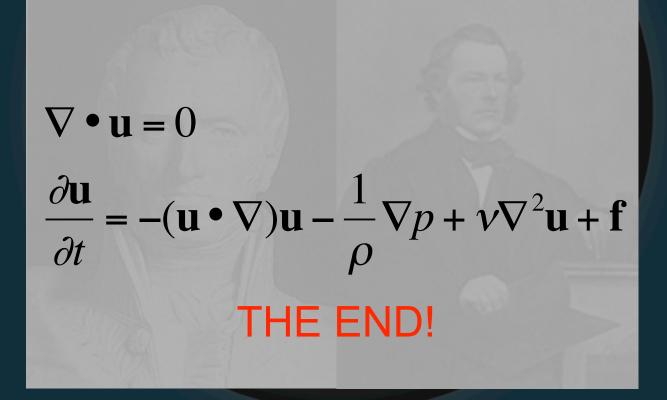


**Mass Conservation** 

#### In principle then, Navier-Stokes is...



#### In principle then, Navier-Stokes is...



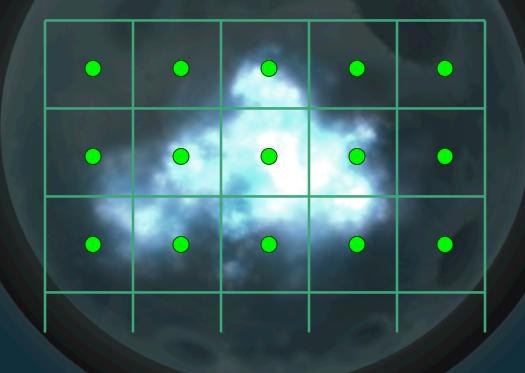
• But not really, of course

- But not really, of course
- Little tiny detail of implementation

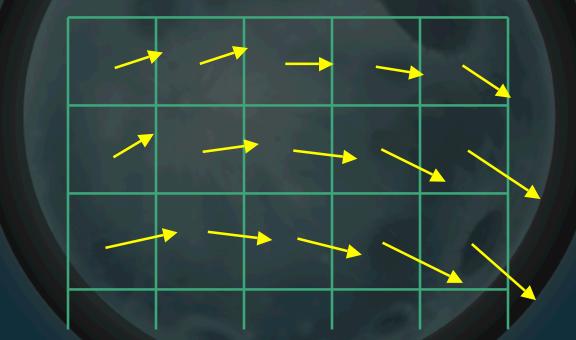
## **Computational Fluid Types**

- Grid-based/Eulerian (Stable Fluids)
- Particle-based/Lagrangian (Smoothed Particle Hydrodynamics)
- Surface-based (wave composition)

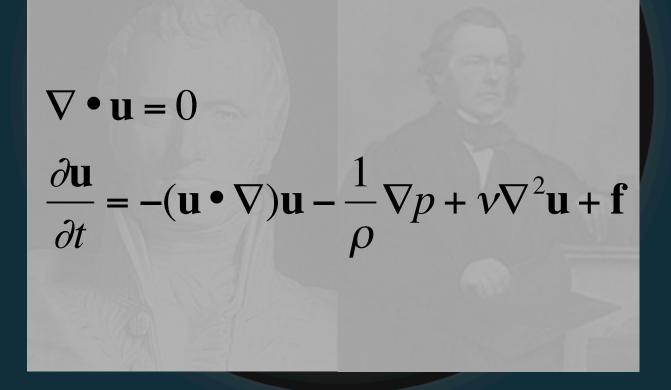
#### Store density, temp in grid centers



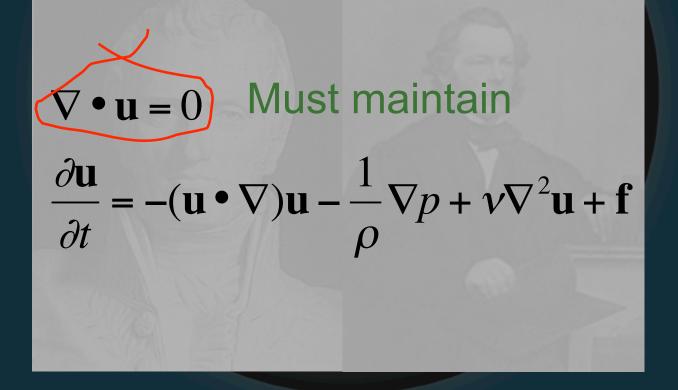
#### Velocity (flow) from centers as well

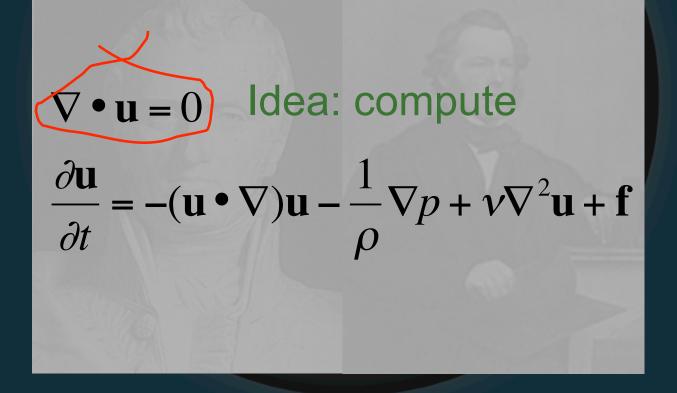


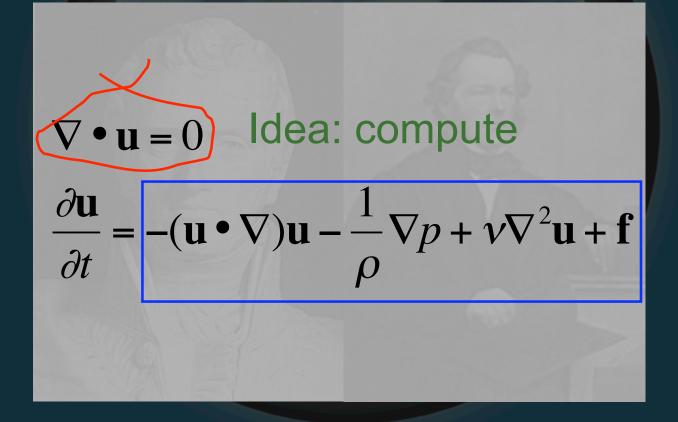
Could also do edges

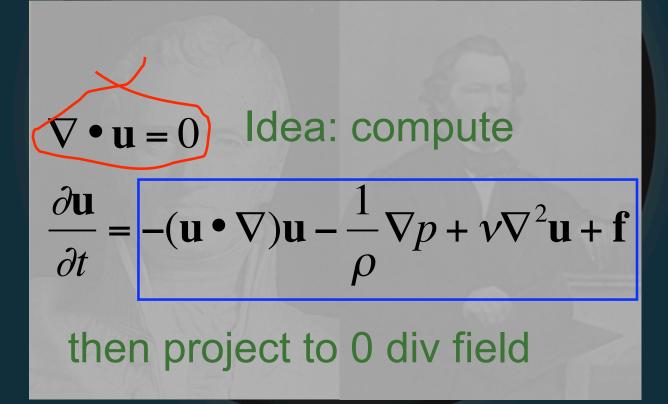


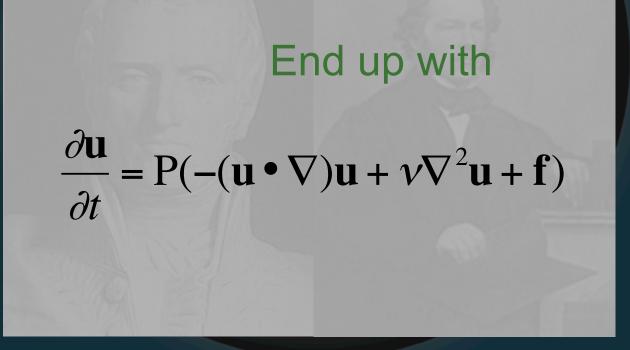
 Jos Stam devised stable approximation: "Stable Fluids", SIGGRAPH '99

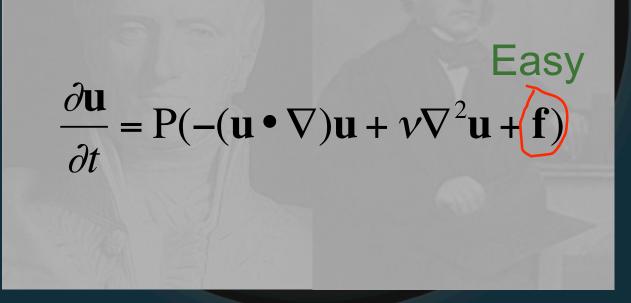


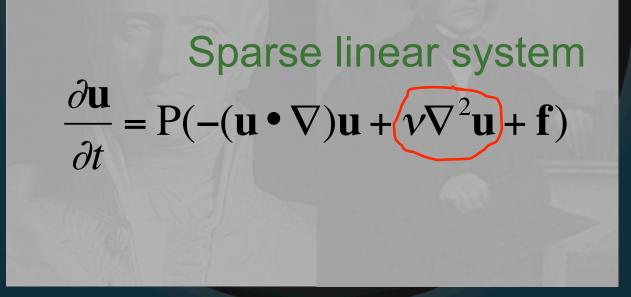


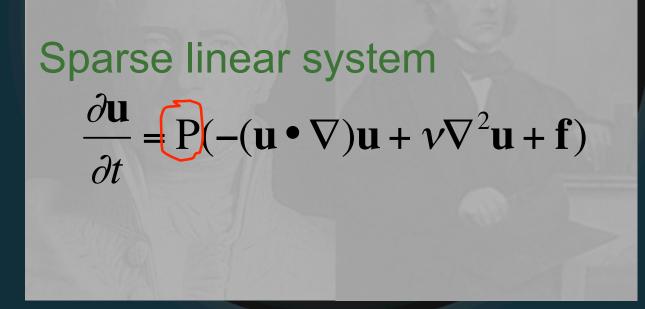


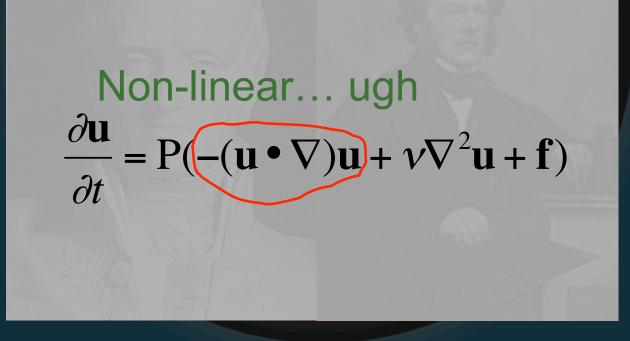








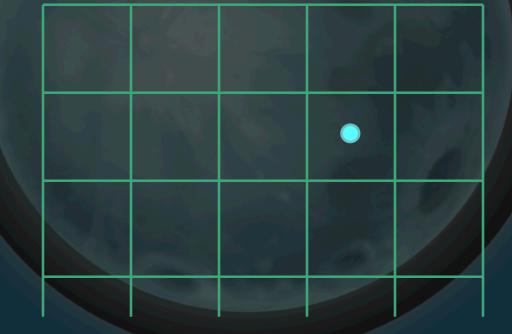




Updating advection
 General idea:

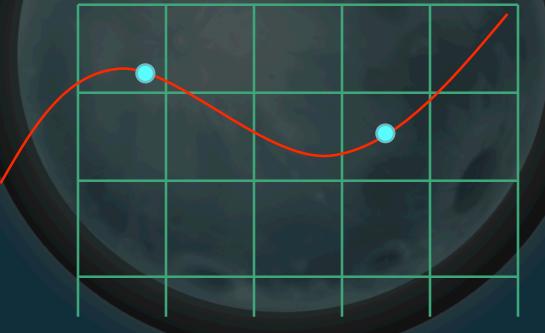
Updating advection

General idea: look at current position



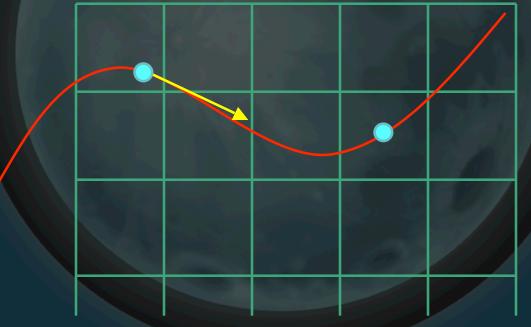
Updating advection

General idea: follow flow to prev. position



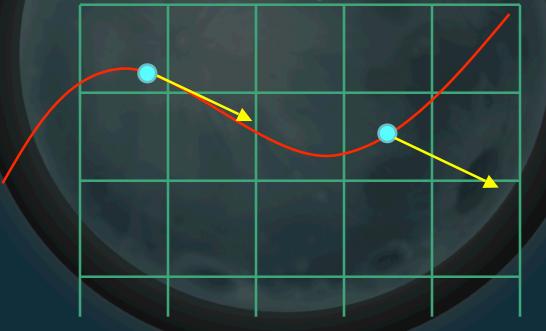
Updating advection

General idea: get velocity there



Updating advection

General idea: assign to current position

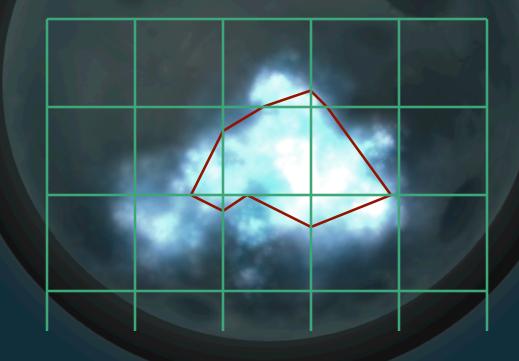


#### Overview

- Update velocities based on
  - Forces, then
  - Advection, then
  - Viscosity
- Project velocities to zero divergence
- Update densities based on
  - Input sources
  - Velocity
  - Diffusion (similar to viscosity, sometimes not used)
- Draw it

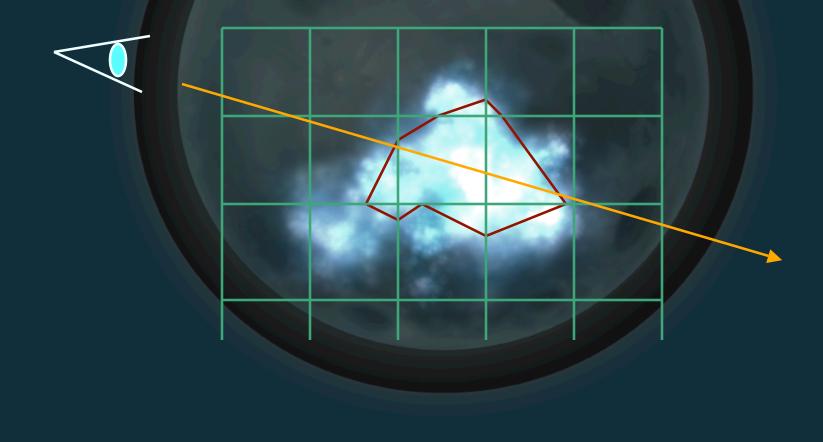
# **Rendering Grid-Based**

Build level surface



# **Rendering Grid-Based**

Determining color, transparency



#### Issues

- Limited space
- Water "splashes" get lost
- Can be computationally expensive
- Dampens down
- But stable

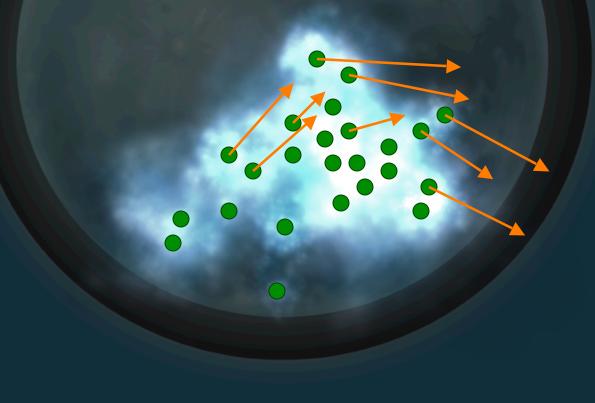
## Implementation

- Little Big Planet
  - "Death smoke"
  - Bubble pop
  - Other smoke effects
- Hellgate: London
- GDC09 NVIDIA demo

# Smoothed Particle Hydrodynamics

 Approximate fluid with small(er) set of particles

#### Velocities at particles provide flow



#### Idea: treat as particle system

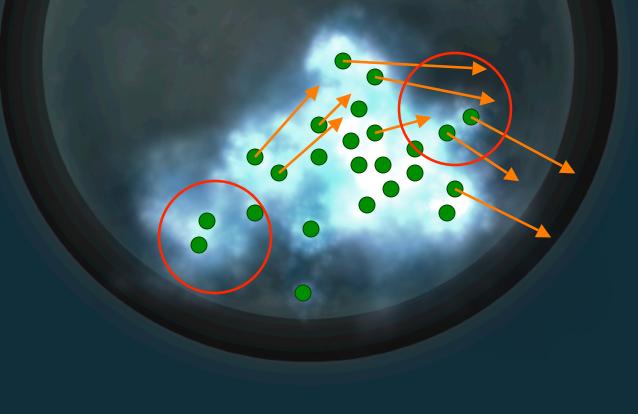
Idea: treat as particle system
 Determine forces

Idea: treat as particle system

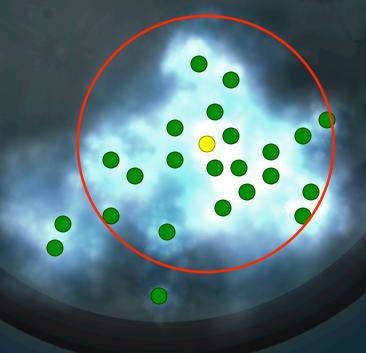
 Determine forces
 Update velocities, positions

 Weighted average gives density (smoothing kernel)

### Can also use kernel to get general velocity



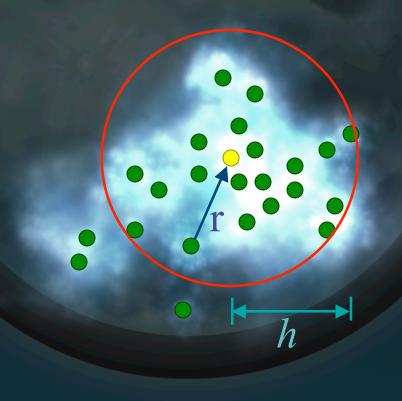
### Usually center at particle

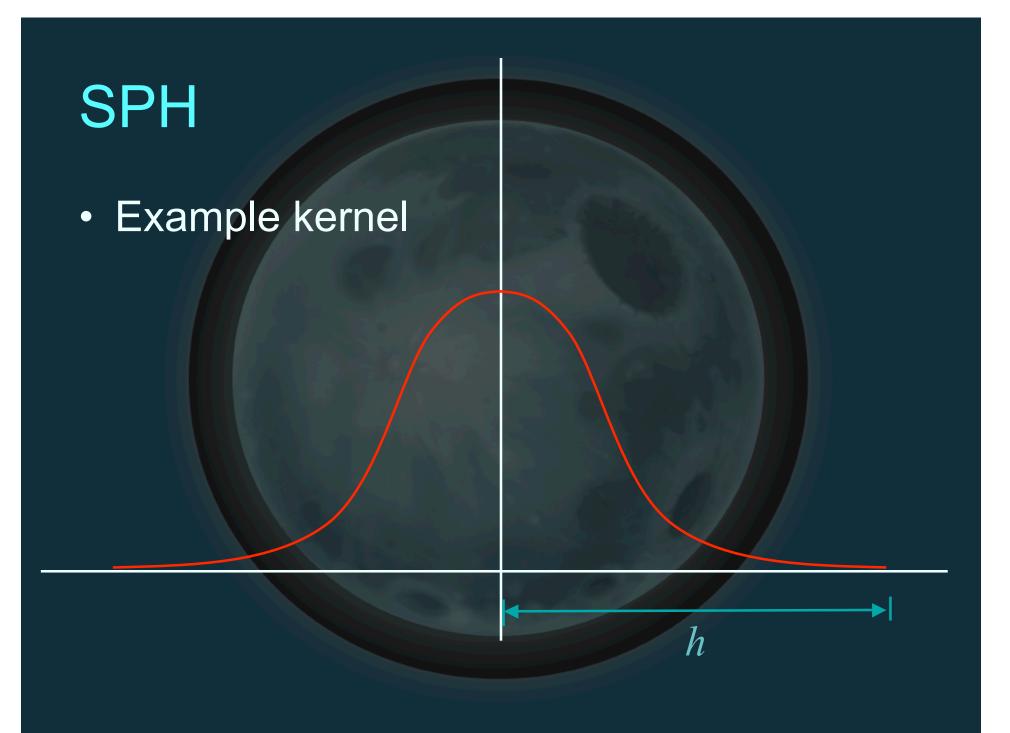


• Specify width by h

h

### Specify vector from other particles by r





Common kernel

$$W_{\text{poly}\,6}(\mathbf{r},h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - r^2)^3 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$

Common kernel

W<sub>poly6</sub>(**r**,h) = 
$$\frac{315}{64\pi h^9} \begin{cases} (h^2 - r^2)^3 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$

#### Clamps to zero at boundary

Common kernel

W<sub>poly6</sub>(**r**,h) = 
$$\frac{315}{64\pi h^9} \begin{cases} (h^2 - r^2)^3 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$

Clamps to zero at boundary Uses length squared

General SPH rule

 $A_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{A_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$ 

General SPH rule

$$A_{s}(\mathbf{x}) = \sum_{j} m_{j} \frac{A_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

### Particle mass

#### General SPH rule

$$A_{S}(\mathbf{x}) = \sum_{j} m_{j} \overbrace{\rho_{j}}^{A_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

### Quantity at particle j

General SPH rule

$$A_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{A_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

### Density at particle *j*

#### General SPH rule

$$A_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{A_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

### Weighting function

### Computing density

 $\rho_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{\rho_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$ 

### Computing density

 $\rho_{S}(\mathbf{x}) = \sum_{i} m_{j} W(\mathbf{r} - \mathbf{r}_{j}, h)$ 

Local pressure

 $p_i = k\rho_i$ 

### k is gas constant

Local pressure

 $p_i = k\rho_i$ 

#### k is gas constant

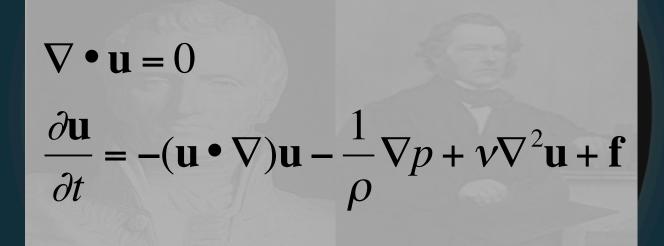
Can be unstable, so...

### Local pressure (alternative)

 $p_i = k(\rho_i - \rho_0)$ 

#### No effect on gradient, more stable

Back to Navier-Stokes

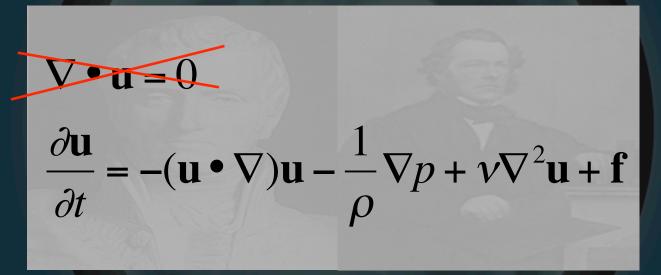


Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$
  
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

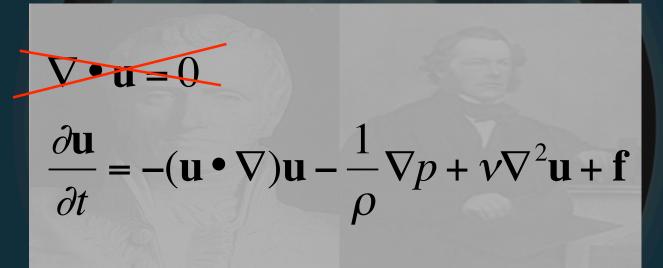
Have fixed # particles and mass, so...

### Back to Navier-Stokes



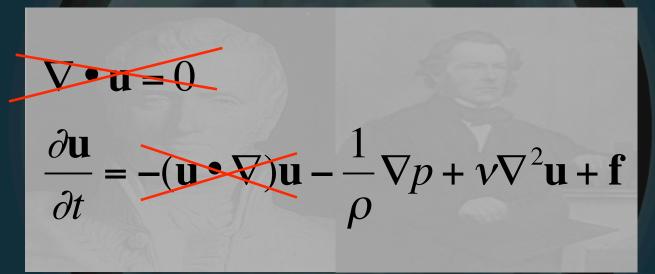
Have fixed # particles and mass, so... mass is automatically conserved

#### Back to Navier-Stokes



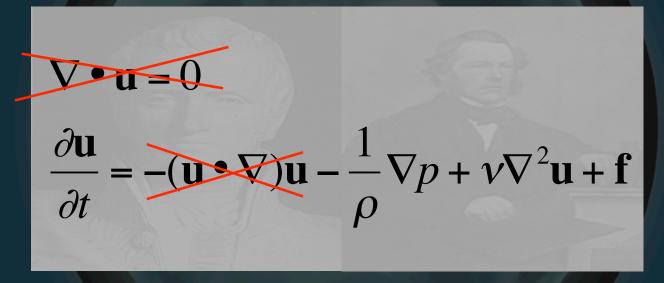
Advection automagically handled by particle update, so...

### Back to Navier-Stokes



Advection automagically handled by particle update, so...

#### Simplifies to



### Simplifies to

 $\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$ 

### Functional breakdown

 $\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$ 

### Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

### Change in velocity

### Functional breakdown

 $\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$ 

#### Pressure

### Functional breakdown

 $\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{u} \nabla^2 \mathbf{u} + \rho \mathbf{f}$ 

Viscosity

### Functional breakdown

 $\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$ 

#### **External forces**

- Compute densities, local pressure
- Generate forces on particles
  - External
  - Pressure
  - Viscosity
- Update velocities, positions
- Render

• Pressure

$$\mathbf{f}_i^{pressure} = -\nabla p(\mathbf{r}_i)$$

#### Pressure

 $\mathbf{f}_{i}^{pressure} = \sum_{j} m_{j} \frac{p_{j}}{\rho_{j}} \nabla W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$ 

• Pressure

$$\mathbf{f}_{i}^{pressure} = \sum_{j} m_{j} \frac{p_{j}}{\rho_{j}} \nabla W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

### Asymmetric, so...

Pressure

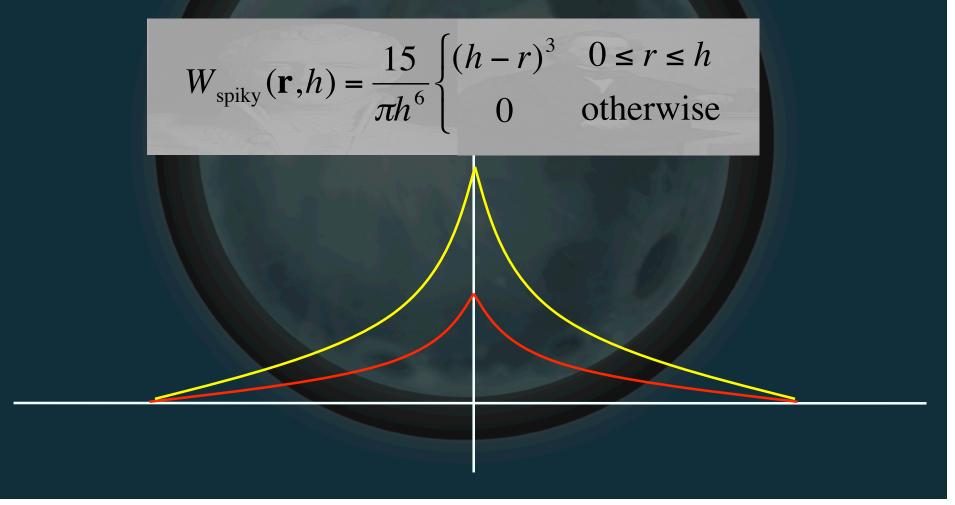
$$\mathbf{f}_{i}^{pressure} = \sum_{j} m_{j} \frac{p_{i} + p_{j}}{2\rho_{j}} \nabla W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

### **Ensures 2-particle interaction equal**

Pressure kernel
 Problem: gradient of poly6 kernel is zero at origin

Pressure kernel
 Problem: gradient of poly6 kernel is zero at origin

Pressure kernel



Viscosity

 $\mathbf{f}_i^{\text{viscosity}} = \mu \nabla^2 \mathbf{v}(\mathbf{r}_i)$ 

#### Viscosity

 $\mathbf{f}_{i}^{\text{viscosity}} = \mu \sum_{j} m_{j} \frac{\mathbf{v}_{j}}{\rho_{j}} \nabla^{2} W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$ 

Viscosity

$$\mathbf{f}_{i}^{\text{viscosity}} = \mu \sum_{j} m_{j} \frac{\mathbf{v}_{j}}{\rho_{j}} \nabla^{2} W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

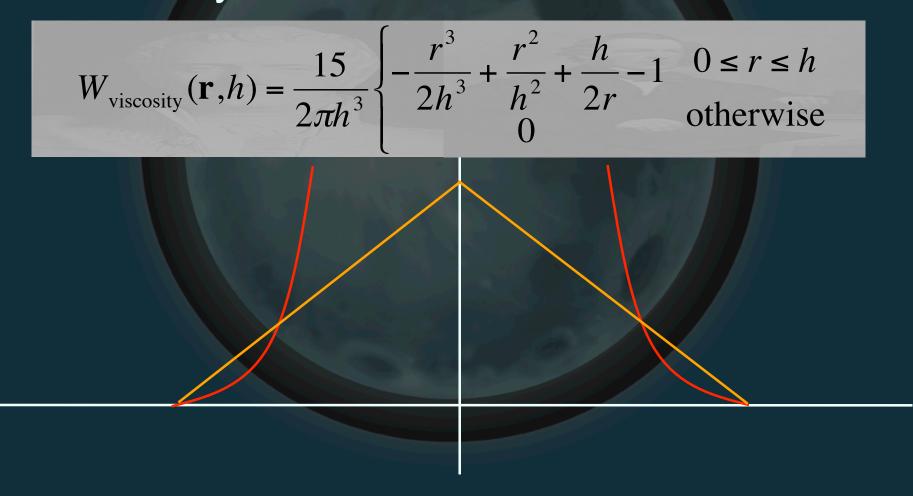
#### Also asymmetric, so...

Viscosity

$$\mathbf{f}_{i}^{\text{viscosity}} = \mu \sum_{j} m_{j} \frac{\mathbf{v}_{j} - \mathbf{v}_{i}}{\rho_{j}} \nabla^{2} W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

#### **Ensures 2-particle interaction opposite**

Viscosity kernel



- Rendering
  - Marching cubes (using smoothing kernel)
  - Blobs around particles/splatting

### **SPH Implementations**

- Takahiro Harada
- Kees van Kooten (Playlogic)
- NVIDIA PhysX
- Rama Hoetzlein\* (SPH Fluids 2.0)
- <u>Takashi AMADA\*</u>

\* Source code available

### **SPH** Issues

- Need a lot of particles
- Computing level surface can be a pain
- Can be difficult to get stable simulation

## **SPH Improvements**

- Spatial hashing
- Variable kernel width
- CFD/SPH Hybrid

   CFD manages general flow
   SPH "splashes"

### **Surface Simulation**

- Idea: for water, all we care about is the airwater boundary (level surface)
- Why simulate the rest?
- This is what Insomniac R20 system does

Done by Mike Day, based on *Titanic* water
 Basic idea: convolve sinusoids procedurally

 Much cheaper to multiply in frequency domain and do FFT (assuming periodic)

Review

- Sinusoid in spatial domain

Review

 Can represent as magnitude+phase in frequency slot

- Review
  - Requires periodic function

FFT

Review

- Multiple sinusoids end up at multiple entries

FF

Wave speed dependant on wavelength

 I.e. phases update at different rates
 AKA dispersion

General procedure

Start with convolved data in (r,φ) form
Update phase angles for each sinusoid
Angular velocity\*dt
Dependent on frequency
Do inverse FFT to get spatial result

- FFT kernel limited to 32x32
- Combine multiple levels via LOD height field scheme
  - Gives high detail close to camera

- Interactive waves
  - Just adding in splashes looks fake
  - Instead, do some more FFT trickery so all our work occurs in the same domain
  - Non-periodic, so have to manage edges
  - Gives nice dispersion effects

- Rendering
  - Rendered as height field mesh
  - Add normal map for detail
  - Cube map/frame buffer map for reflections
  - Distortion effect for refractions

• <u>Nifty video</u>

#### References

- Jos Stam, "Stable Fluids", SIGGRAPH 1999
- Mattias Müller, et. al, "Particle-Based Fluid Simulation for Interactive Applications", SIGGRAPH Symposium on Computer Animation 2003
- Jerry Tessendorf, "Simulating Ocean Water," SIGGRAPH Course Notes.
- http://www.insomniacgames.com/tech